

On visual prior entry

by

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## Curriculum Vitae

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## Abstract

The prior entry hypothesis contends that attending to a stimulus shortens its time to perception. Historical support for the hypothesis is weak, but the results of recent temporal order experiments employing exogenous visual cues purportedly support prior entry by demonstrating robust shifts in the point of subjective simultaneity (PSS) between attended and unattended stimuli. An alternative hypothesis to prior entry is that attention influences the decision process.

In a series of experiments, observers made judgments of simultaneity (SJ) or temporal order (TOJ) for two stimuli, to one of which attention was oriented by exogenous, endogenous, gaze-directed, isoluminant exogenous or multiple exogenous cues. All types of cues, at appropriate lead times, caused shifts in the PSS in the TOJ, but a decision model that explicitly includes response biases could account for the shift in the PSS of the TOJ without the inclusion of an additional attentional acceleration parameter. Therefore, observed shifts caused by attention of the PSS in a TOJ cannot alone be used to accept the prior entry hypothesis. The shifts in the PSS revealed by the SJ, which is less prone to response biases, were generally smaller and were only significantly evoked by abrupt onset cues at the target location. Using multiple exogenous cues to dilute the attentional allocation of any single cue revealed that the effect of an exogenous cue at a short cue lead time is composed of both an attentional and a non-attentional component. Thus, the probable scope of the prior entry hypothesis is quite limited.

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## List of Recurring Symbols and Abbreviations

Symbol or abbreviation	Description	Page of definition
$\mathbf{A}$	Hessian matrix of the log-likelihood function	65
$\alpha$	Latency reduction due to attention	56
$\hat{\alpha}$	Maximum likelihood estimator of $\alpha$	80
$\beta$	Response bias parameter	61
$\hat{\beta}$	Maximum likelihood estimator of $\beta$	80
$\chi_N^2$	Chi-square distribution with $N$ degrees of freedom	66
CLT	Cue lead time	27
$\mathcal{D}$	Experimental data	64
$\delta$	Delay between stimulus onset and central arrival	56
$\Delta L$	Difference in latencies of the two targets	56
$\Delta t$	The SOA between the two stimuli	30
EEG	Electroencephalogram	16
ERP	Evoked response potential	16
$h$	Height of the scaled normal distribution	30
$\mathcal{H}_0$	The null hypothesis: $\alpha = 0$	65
$\mathcal{H}_1$	The alternative hypothesis: $\alpha \neq 0$	65
$\mathcal{H}_4$	Four parameter joint SJ and TOJ hypothesis	79
$\mathcal{H}_5$	Five parameter joint SJ and TOJ hypothesis	78
$\mathcal{H}_7$	Sever parameters independent SJ and TOJ hypothesis	78
$\mathcal{H}_k$	A non-specific hypothesis $k$	63
$\mathcal{L}$	Likelihood function	64



$L_1$	Central arrival latency of Stimulus 1	56
$L_2$	Central arrival latency of Stimulus 2	56
$\mathcal{L}_{\max}$	Maximum of the likelihood function	64
$\lambda$	Ratio of likelihood functions	65
$m$	Number of conditions	64
$\mu$	Mean of a normal distribution	30
$\mu_1$	Mean arrival time for $S_1$	56
$\mu_2$	Mean arrival time for $S_2$	56
$\mu_{sL}$	Mean arrival time difference between $S_1$ and $S_2$	56
$N$	Number of observers	30
$\mathcal{N}$	Normal probability distribution function	56
$\mathcal{N}^+$	Shorthand notation for $\mathcal{N}(\tau, \Delta t + \alpha, \sigma)$	98
$\mathcal{N}^-$	Shorthand notation for $\mathcal{N}(-\tau, \Delta t + \alpha, \sigma)$	98
$\mathcal{N}^0$	Shorthand notation for $\mathcal{N}(0, \Delta t + \alpha, \sigma)$	98
$n_i$	Number of repetitions for condition $i$	64
$p_i$	Probability of response $R_0$ for condition $i$	64
$\phi$	Scaled normal probability distribution function	30
$\Phi$	Cumulative normal distribution function	60
$\Phi^+$	Shorthand notation for $\Phi(\tau, \Delta t + \alpha, \sigma)$	98
$\Phi^-$	Shorthand notation for $\Phi(-\tau, \Delta t + \alpha, \sigma)$	98
$\Phi^0$	Shorthand notation for $\Phi(0, \Delta t + \alpha, \sigma)$	98
PSS	Point of subjective simultaneity	6
$R_0$	“Simultaneous” or “ $S_1$ first” response	64
$R_1$	“Successive” or “ $S_2$ first” response	64
$S_1$	Stimulus 1, the attended stimulus	1
$S_2$	Stimulus 2, the unattended stimulus	1
$s$	Sample standard deviation of the weighted average	30

$\sigma$	In general, the standard deviation of a normal distribution. Often refers to $\sigma_{sL}$ .	57
$\hat{\sigma}$	Maximum likelihood estimator of $\sigma$	80
$\sigma_1$	The width of the latency distribution for $S_1$	56
$\sigma_2$	The width of the latency distribution for $S_2$	56
$\sigma_{sL}$	Width of the latency difference distribution	56
SJ	Simultaneity judgment	1
SOA	Stimulus onset asynchrony	6
$t_1$	Onset time for $S_1$	1
$t_2$	Onset time for $S_2$	1
$t_{N-1}$	Student $t$ distribution with $N-1$ degrees of freedom	31
$\tau$	Simultaneity decision criterion	59
$\hat{\tau}$	Maximum likelihood estimator of $\tau$	80
TOJ	Temporal order judgment	1
VEP	Steady-state visual evoked potential	17
$\mathbf{w}$	Vector of model parameters	63
$w_k$	Weighted average weight for observer $k$	31
	Also, scalar component of $\mathbf{w}$	97
$\bar{x}$	Sample weighted average of a parameter across observers	30
$x$	CIE color coordinate	26
$X_{ij}$	Response on repetition $j$ of condition $i$	64
$\xi^+$	Shorthand notation for $\tau - \Delta t - \alpha$	98
$\xi^-$	Shorthand notation for $-\tau - \Delta t - \alpha$	98
$\xi^0$	Shorthand notation for $-\Delta t - \alpha$	98
$y$	CIE color coordinate	26
$Y_i$	Number of $R_0$ responses for condition $i$	64

# I Introduction

The properties of attention have been studied since from the very beginnings of experimental psychology. Researchers generally agree that attending to a region of visual space facilitates the perceptual discriminability of and the behavioral reaction time to items in that region. Another potential property remains controversial. In the spirit of the successful laws of physics of the time, the American psychologist Edward Titchener in 1908 codified what was known about the properties of attention into a series of laws. Among these is what he called the law of prior entry: an attended stimulus reaches consciousness before an otherwise identical but unattended stimulus. Although the support for prior entry was questionable even at that time, the notion that attention might accelerate processing managed to become widespread.

Unlike the behavioral reaction time to the appearance of an item in the visual field, which can be measured in absolute terms, the latency to perception, which has no known physical correlates, may at this time only be measured in relative terms. Given two stimuli,  $S_1$  and  $S_2$ , occurring at times  $t_1$  and  $t_2$ , an observer can report which was perceived first, or whether the two stimuli appeared simultaneously or successively. These types of judgments are respectively known as temporal order judgments (TOJ) and simultaneity judgments (SJ). Recently a number of TOJ experiments within the visual domain have apparently provided strong support for the prior entry theory, but only for certain types of attentional cueing.

This dissertation consists of five parts, including this introduction. In Part II, I briefly review the literature of prior entry, and develop two competing hypotheses that might account for the results: the attentional acceleration hypothesis contends that attending to a stimulus reduces its transmission latency to the central decision mechanism; attentional bias hypothesis contends that attention does not affect transmission latency but instead alters response criteria in the decision process. In Part III, I report a number of experiments performed to replicate current results under controlled conditions and to provide further data that might be able to discriminate

between the hypotheses. In Part IV, I describe a possible mechanisms for the decision process, and in Part V, I use the data from Part III and the model from Part IV to evaluate the two hypotheses.

## II Background

### II.1 History of prior entry

Titchener's fourth of his seven laws of attention is the law of prior entry, which he summarized as, "The object of attention comes to consciousness more quickly than the objects which we are not attending to" (Titchener, 1908, p. 251). The prior entry theory was based on experiments that began some ninety years before, and it has been investigated for some ninety years since without receiving very convincing support until recently. Studies of attention and perceptual latency can be classified historically into four distinct phases. In the first phase, experiments purporting to show the prior entry effect of attention were plagued by poor methodology and employed attention only as a post-hoc explanation of the results. The second phase of experiments manipulated attention with instructions, but the results were mixed. In the third phase, researchers tried to control attention more strictly by forcing observers to use the information presented in one of the stimuli, or by manipulating frequency or payoffs. Even so, the prior entry effect was not always observed, and even when it was, alternative explanations seemed at least as likely as an attentional acceleration. Until the last decade, the preponderance of evidence did not support the claim that attention affected temporal order judgments at all, much less that it caused apparent prior entry effects through an acceleration of sensory processing. The fourth and current phase of the history of prior entry includes experiments that use attentional cuing paradigms entirely within the visual domain, compared to the previously typical multimodal paradigm, and only now have reports of prior entry been consistently replicated.

### II.1.1 Complication experiments

Titchener's (1908) laws were primarily statements about what was known about attention at the time, and the law of prior entry was stimulated by the results of "complication" experiments in which observers judged the position of a moving visual object at the instant an auditory stimulus sounded. The formation of a coherent experience was thought to involve the combination or complication of the two stimuli. Boring (1929) describes the prior entry theory as being the result of work accomplished by von Tschisch, Pflaum and Geiger in Wundt's Leipzig Laboratory during 1885–1902. These experiments were motivated by the field of astronomy, in which what Bessel (1822) termed a "personal equation" was found to be necessary to calibrate measurements of stellar transit times among observers (see Sanford, 1888, for a review).

While the complication experiments spurred research into the question of prior entry, they provide little support for the theory. First, these experiments employed inadequate methodology. Dunlap (1910) showed that the results of the complication experiments depended in large part on eye movements and fixation. Second, attention was merely one of many possible post hoc explanations. For example, Cairney (1975b) suggested that judgment strategy was a better explanation than prior entry for his version of the complication experiment using multiple directions of movement.

### II.1.2 Direction of attention using instructions

In the early complication experiments, although attention was thought to cause the results, it was not manipulated directly. In later experiments, control of attention was attempted by instructions encouraging the observers to pay attention to a modality or expect a stimulus. However, the prior entry effect was not always found in such experiments. For example, neither Hamlin (1895) nor Drew (1896) found any

relation between the direction of attention and the order of pairs of stimuli from different modalities. Other results were more successful. For a concise demonstration of prior entry, Titchener cited Stevens' (1904) variation of the complication experiment described by Wundt (1874). While observing a metronome whose bell rang when its pendulum passed a certain point, Stevens reported that bell was heard either before or after the pendulum reached that point, depending on whether he was paying attention to the bell or to the pendulum. In Stone's (1926) experiments, observers judged the order of auditory and tactile stimuli pairs while being told to expect and attend to either the sound or touch. Observers tended to report that the attended stimulus occurred first, and the means of the psychometric functions over various interstimulus intervals were shifted in favor of the attended modality. Needham (1934; 1936) demonstrated an apparent prior entry effect for a buzzer sounding among a series of five other tones. Although the observers were instructed to expect the buzzer, they agreed that attending to the buzzer was not possible, and some specifically reported performing the task by listening to the entire series of tones and then trying to retroactively place the buzzer within the series.

### II.1.3 More reliable attentional allocation

Later experiments attempted to direct attention through more reliable means, other than simple instruction or expectation. Sternberg, Knoll and Gates (1971) presented auditory-tactile or auditory-visual stimuli pairs and required observers to make a speeded judgment to stimuli presented in the modality that had been cued followed by a judgment of temporal order. The majority of observers exhibited a clear shift of the point of subjective simultaneity favoring the attended modality—the prior entry effect. Vanderhaeghen and Bertelson (1974) directed attention to a sound or light through frequency and payoff manipulations. Even though these manipulations decreased reaction times to these stimuli, they did not affect the apparent temporal ordering with a reference tactile stimulus. Cairney (1975a) required observers to

discriminate a feature of either the auditory or visual stimulus in an attempt to direct their attention without using explicit attentional instructions. However, directing attention in this manner produced no effect on the perceived temporal order of the two types of stimuli.

Frey's (1990) experiments with auditory and visual stimuli suggested that the prior entry effect was mainly due to cognitive factors rather than a change in perceptual latency. Attention was directed by requiring the observers to make speeded responses and judgments to stimuli in one of the modalities. When observers were instructed to report which of two stimuli occurred first (Experiment 3), a prior entry effect occurred for stimuli in the attended modality. However, when observers were asked to report which stimulus occurred second (Experiment 7), a prior entry effect now occurred for the unattended stimuli. It seemed that attending to a modality simply increased the probability of response to stimuli in that modality, despite whether the task was to judge which came first or second.

## **II.2 Evidence for prior entry with visual attention**

The most recent and most convincing reports of prior entry involve judgments between two visual stimuli and generally employ visual cues to direct attention. These recent experiments generally use a TOJ to show a shift in the point of subjective simultaneity (PSS) as a function of attention. Two stimuli, one attended and one unattended, are presented with some stimulus onset asynchrony (SOA) between them. A psychometric function is determined over a range of SOAs, and the PSS is determined as the SOA at which the psychometric function crosses 50%. At the PSS, the observers are maximally uncertain about the order of the two stimuli. The magnitude of the PSS shift has been found to depend on the type of cue used to orient attention. Before discussing the results, I will briefly review the methods used to covertly orient attention.



### II.2.1 Covert changes of visual attention

Not too long after Sternberg et al. (1971) presented the best evidence to that time that attention caused a prior entry effect, researchers began to investigate cuing techniques to direct the covert orientation of attention (Posner, 1980)—without eye movements—within the visual field (see Yantis, 1998, for a review). An endogenous cue indicates the target location only symbolically, either through instructions or a stimulus remote from the target position, and the observer must orient attention voluntarily (e.g. Posner, 1980; Posner, Snyder & Davidson, 1980). An exogenous cue is typically an abrupt visual onset at or near a peripheral target location (e.g. Eriksen & Hoffman, 1972; Posner & Cohen, 1984). Exogenous cues operate automatically (see LaBerge, 1981), reflexively orienting attention even when known never to indicate the location of a visual target (Remington, Johnston, & Yantis, 1992), though Yantis and Jonides (1990) showed that focused endogenous attention could override exogenous cues to some extent. Both types of attentional cues facilitate accuracy (e.g. Bashinski & Bacharach, 1980; Egly & Homa, 1984; Shaw & Shaw, 1977; van der Heijden & Eerland, 1973) and detection or identification reaction times (e.g. Jonides, 1981; Posner, 1980; Posner et al., 1980).

The dynamics of exogenous and endogenous attention are different. With minor discrepancies between experiments employing a variety of methods, it has been shown that the effects of exogenous cueing are present as early as 25 ms after the cue, reach a maximum around 100–150 ms, and asymptote to a sustained level by 400 ms or less after the cue (Cheal & Lyon, 1991; Müller & Rabbitt, 1989; Murphy & Eriksen, 1987; Nakayama & Mackeben, 1989; Posner & Cohen, 1984). The effects of endogenous cues, on the other hand, take longer to develop, reaching a sustained level by 300–400 ms. The effects of endogenous and exogenous cues are very similar after about 400 ms.

Eye movements are always a concern when measuring the effects of covert attentional orienting, but it has been found that the act of monitoring eye movements

may have some influence on covert attention. Nakayama and Mackeben (1989) found that when eye movements were not monitored, the accuracy was similar to the monitored case until 125 ms after the cue and decreased thereafter. Cheal and Lyon (1991) found that, for one of two observers, discarding trials in which there were eye movements increased accuracy.

## II.2.2 Reports of prior entry with visual attention

Stelmach and Herdman (1991) and Hikosaka, Miyauchi and Shimojo (1993) used TOJs to show that for exogenous cueing, stimuli in the unattended location needed to be presented some 50 ms before those in the attended location in order to be reported equally likely to have appeared first. To attempt to minimize response biases, Stelmach and Herdman used a ternary response task, allowing a third “simultaneous” response in addition to the “attended target first” and “unattended target first” responses. A shift in the PSS was still observed under this condition, but when observers were instructed to attend to one of the two locations (as opposed to the center between the objects), the “simultaneous” response was rarely employed, and it is not clear whether the observers were actually entertaining its use.

There is some controversy whether endogenous cuing also produces shifts in the PSS. Stelmach and Herdman (1991) found a small shift in the PSS for endogenous attention when a binary response was used, but Jaskowski (1993) showed that allowing observers the option of the “simultaneous” response eliminated any temporal order preference for the endogenously cued stimulus.

Zackon, Casson, Zafar, Stelmach and Racette (1999) reported that temporal order judgments were affected by exogenous, and to a smaller degree, endogenous cues. The cues tended to increase the likelihood that the observers would report that a cued stimulus occurred before a simultaneously presented uncued stimulus. The effect of the exogenous cue increased as the delay between the cue and stimulus increased from 0 to 300 ms. They suggested that exogenous cues do not influence the

speed at which information is processed and transmitted, but rather compress the perceived time interval between the cue and stimulus.

Shore, Spence and Klein (2001) attempted to replicate Frey's (1990) findings in the visual domain. They oriented observers' attention either exogenously or endogenously to one of two visual stimuli, and, in separate experiments, asked observers to report which stimulus occurred either first or second. When the observers judged which stimulus came first, their PSSs favored the attended stimuli for both exogenous and endogenous cueing, with larger shifts for exogenous cueing. When the observers judged which stimulus came second, the PSS shifts were reduced but not reversed as in Frey's study. They argued that averaging the PSSs obtained in the two response conditions could factor out response biases, and they concluded that cognitive factors contributed to but did not consume the observed prior entry effects in the visual domain.

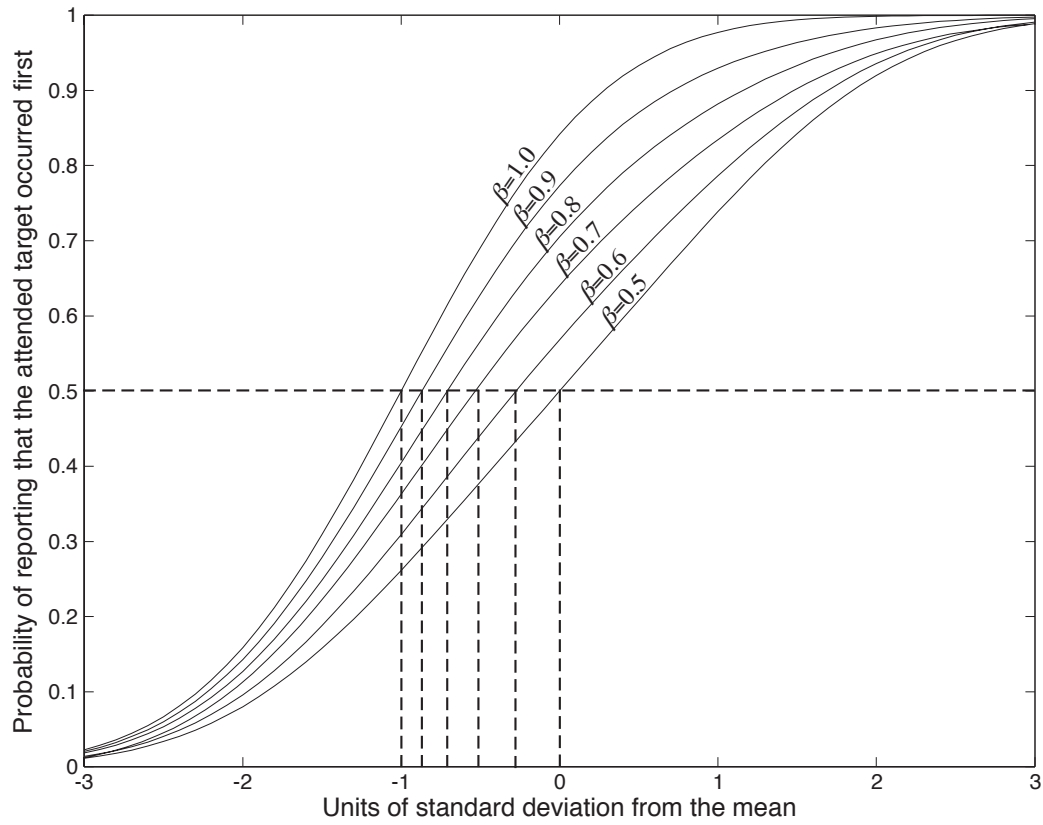
### **II.3 Hypotheses to explain the apparent prior entry effect**

According to the prominent view outlined in Sternberg and Knoll (1973) the decision that determines the perceived temporal order of two stimuli operates on the difference of the independent arrival times of the two stimuli (see Stelmach & Herdman, 1991, for an alternative that does not assume independence). Given that a stimulus is encoded into a series of neural impulses, the measurement of its latency is not trivial. Lennie (1981) discussed several possibilities. One is an integration method that determines time for a criterion number of action potentials to occur. By measuring the latency from recordings in retinal ganglion cells, he showed that as a stimulus flash became brighter, the response latency and variability decreased. Allik and Pulver (1994) investigated the mechanism for latency detection using temporal order judgments between stimuli with different luminance onset durations. For example, one stimulus might have an abrupt onset while the other gradually increased in luminance. They found that shifts in the PSS between such stimuli could be

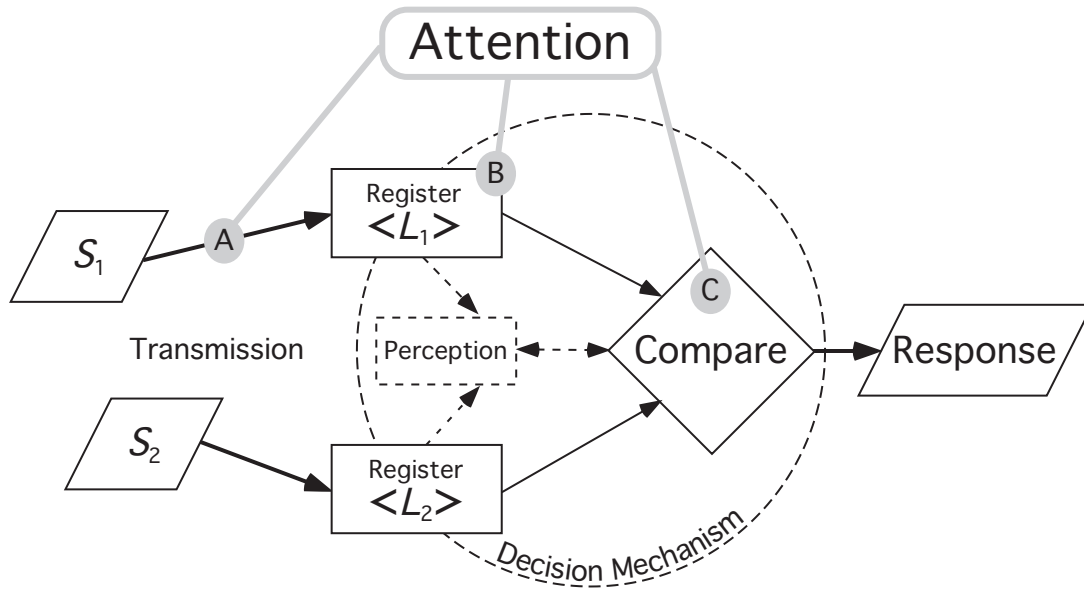
explained by supposing that the latency of each stimulus is determined by the time at which a low-pass filtered version of the stimulus profile exceeded a criterion.

The studies above that claim to demonstrate prior entry generally do so by showing that attention causes a shift in the measured PSS. It is indisputable that exogenous cueing of one stimulus alters the PSS in TOJ experiments, and in addition, there is mixed evidence that endogenous cuing also shifts the PSS, although the effect is much smaller than for exogenous cuing. Two competing hypotheses can explain these observed shifts. The first is the attentional acceleration hypothesis. This hypothesis suggests that attention alters the latencies of the two stimuli before they are registered at the central decision mechanism responsible for generating a response. Hikosaka et al. (1993) suggest that attention accelerates visual processing at a very early stage. They suggest that the acceleration effect can produce a temporal delay between the perception of the attended and unattended sides of a static object, and that this delay that can activate motion mechanisms to produce a sense of motion within the object.

The second hypothesis is the attentional bias hypothesis. This hypothesis contends that attention does not alter the latencies of the stimuli, but rather the decision process that operates on the difference of these latencies. One possibility, as argued by Jaskowski (1993; also see Spence & Driver, 1997), is that attention produces response biases favoring attended stimuli. For example, if the temporal delay between the onsets of two stimuli is too small to permit accurate temporal order discrimination, observers might simply guess in a manner biased by a number of factors, among them their attentional state. An example of the shift in the PSS of a psychometric function entirely due to response bias is illustrated in Figure 1. The function shown is an example of the triggered-moment model, one of the several decision mechanisms outlined by Sternberg and Knoll (1973), modified to include response bias.



**Figure 1.** Response biases yield shifts in the point of subjective simultaneity, the point at which the function crosses 50%. An example of the triggered-moment temporal order judgment function (Equation 6) is shown with  $\alpha = 0$ ,  $\sigma = \tau = 1$ , and various levels of response bias.  $\beta = 0.5$  indicates a lack of response bias, and  $\beta = 1$  indicates that the observer always responds, when uncertain, that the attended target occurs first.



**Figure 2.** Flowchart of the stimulus-response process for a temporal order judgment. Attention can affect the judgment of the temporal order of two stimuli,  $S_1$  and  $S_2$ , at various stages during the stimulus-response process. A. Attention could accelerate the transmission of one stimulus relative to the other. B. Similarly, attention could influence the measurement of the latency of the stimulus to central decision mechanism, e.g. by reducing the detection criterion. C. Attention could influence the comparison between the two latencies  $L_1$  and  $L_2$ , perhaps causing  $S_1$  to be reported first even if  $L_1 \geq L_2$ . It is not clear what role perception might play in the decision mechanism. The attentional acceleration hypothesis contends that attention operates only at points A or B, while the attentional bias hypothesis contends that attention operates only at point C.

The differential loci of attentional action between the two hypotheses are illustrated in Figure 2. Both of these hypotheses can account for reductions in reaction time due to attention and the attentional effects on temporal order judgments, but they make different predictions as will be described below. These are operational hypotheses that do not mention perception or consciousness, the observer of the classic prior entry hypothesis, but only behavioral decision processes. At present, the neural correlates of conscious perception have not been adequately characterized, and

it is not possible to measure perception independently of behavioral processes. If it can be assumed that conscious perception precedes rather than being the result of the central decision process, then the attentional acceleration hypothesis is identical to the proper prior entry hypothesis that states that attention to a stimulus reduces its latency to consciousness.

The distinction between the attentional acceleration and attentional bias hypotheses parallels a debate in the literature about the general affects of attention. Some researchers feel that attention affects the quality of perceptual processing (e.g. Bashinski & Bacharach, 1980; Downing, 1988; Handy, Kingstone & Mangun, 1996; Luck et al., 1994; Posner et al., 1980), while others believe that attention only affects changes in the decision process (e.g. Müller & Findlay, 1987; Shaw, 1984). Decreases in reaction time due to attention could be the result of increased evidence available to the decision process, or attention could alter the decision process to accept less evidence from likely locations. Under the attentional bias hypothesis, attention can still affect stimuli early in the visual processing hierarchy, but only on properties orthogonal to the latency of the stimulus. The prominent view suggests that attention operates as a gain control mechanism (see Hillyard, Vogel & Luck, 1998, for a review), but changes in the amplitude of the stimulus response may or may not be transformed into latency changes, depending on the nature and dynamics of the decision mechanism.

## **II.4 Reaction time and response bias**

Several researchers have compared reaction time and TOJs to see whether they could be used as measurements of the same perceptual processes. Rutschmann and Link (1964) and Gibbon and Rutschmann (1969) tried to predict the results of a temporal order judgment between two stimuli based upon the distribution of reaction times to those two stimuli. They found that the position of the predicted TOJ function did not match the actual results, but that the shapes of the predicted and actual functions were

similar. They suggest that the difference in position was due to biased response criteria in the TOJ and that the similar shapes indicated that the variance added by the motor component in the reaction time task was low.

Neumann, Esselmann and Klotz (1993) compared reaction time and TOJs with exogenously cued targets. Reaction time was 36 ms faster for cued targets than for uncued, and the PSS in the TOJ was shifted by 26 ms in favor of the cued target. In another experiment, replicated by Steglich and Neumann (2000), masked cues, unreported and apparently unperceived by the observers, were found to reduce reaction time by 29 ms but had little effect on the TOJ. They suggest that cues have a larger effect on response parameters than on perceptual processes. Tappe, Nepal and Neumann (1994) found that spatial frequency also affected reaction times much more than TOJs. They suggest that conscious perception is required for TOJs, but not necessarily for simple motor reactions, consistent with the functional difference between a judgment and a simple response outlined by Ansorge, Klotz and Neumann (1998).

## **II.5 Physiological data**

There are a number of physiological experiments with results bearing on the two hypotheses. If attention accelerates visual processing at one stage of processing, then a latency reduction should be observable in neural activity during subsequent stages. Although the strictness of the functional hierarchy in the visual cortex is often overstated, for the purposes of this discussion, the processing stage hierarchy can be ranked in terms of the latency of the earliest response to a stimulus. If attention does not affect the latency of response in one visual area, perhaps only causing an amplitude change in that area, then the attentional acceleration effect must be confined only to subsequent analysis stages. In this way a temporal window of operation for the attentional acceleration effect can be delimited.



The dependence of the latency on luminance is a good test case that illustrates a latency effect at early processing stages and its effect at later stages. There is a sound body of work, employing a number of different methodologies, showing that stimuli of lower luminance have longer perceptual latencies. This has been shown using a SJ (Roufs, 1963; Roufs, 1974), TOJ (Rutschmann, 1973), by observing the perceptual phase lag between two oscillating stimuli of different luminances (Wilson & Anstis, 1969), and a number of experiments on the Pulfrich (e.g. Carney, Paradiso & Freeman, 1989; Julesz & White, 1969; Nickalls, 1986; Prestrude, 1971; Pulfrich, 1922; Williams & Lit, 1983) and Hess (e.g. Hess, 1904; Williams & Lit, 1983) effects. Roufs (1974) showed that as luminosity ranged over four orders of magnitude, latency, as determined both by reaction times and SJs, changed by 50 ms. Latencies measured by reaction times or TOJs deviated only at the lowest luminosities. Physiological experiments have shown that reaction times and VEP latencies increase in parallel with increases in luminance (Jaskowski, Pruszevicz & Swindzinski, 1990; Vaughan, Costa & Gilden, 1966; Wilson & Lit, 1981). Gawne, Kjaer and Richmond (1996) showed that latency (as defined by the time at which the response to a stimulus reached half its peak) in V1 depended on contrast similarly as retinal ganglion cells depend on luminance, varying by 30–40 ms over the contrast range.

For endogenously cued attention (usually sustained by instructions and by having monkeys perform a task at the attended location), it is possible to rule out latency effects that take place very early in the cortical hierarchy. Single unit recordings in areas MT (e.g. Treue & Maunsell, 1996; Treue & Maunsell, 1999), V2 and V4 (e.g. Luck, Chelazzi, Hillyard & Desimone, 1997; McAdams & Maunsell, 1999) show attentional gain changes, but the onset times of the responses did not seem to be affected by attention. In some cases the gain changes were evident early in the stimulus response, as the attended response began at the same time as the unattended response but increased somewhat more steeply. In other cases, profiles were identical, and the attended and unattended responses diverged only after some

time. Another mechanism of attention to a region of space might be to increase the baseline firing rate of neurons with receptive fields in that region (Luck et al., 1997). Seidemann and Newsome (1999) used an exogenous cue with a lead time of 700 ms, beyond the time range when exogenous cues are most effective. They reported that, in area MT, the time course for the attended and unattended responses were initially similar but diverged after about 250 ms.

Similarly, attention appears to modulate amplitudes but not the latencies of the early electroencephalogram (EEG) components (Mangun, 1995). A number of researchers (e.g. Gomez Gonzalez, Clark, Fan, Luck & Hillyard, 1994; Luck et al., 1994; Mangun, Hansen & Hillyard, 1987; Mangun & Hillyard, 1991) have shown that endogenous attention affects the amplitudes of early evoked response potential (ERP) components that seem to arise from extrastriate cortex. Any conclusions drawn from ERP latencies must be qualified by their interpretation difficulties. The ERP can be caused by a number of sources whose signals can combine in complex ways. An apparent change in the peak latency of one component could be due to additions or subtractions from an amplitude changes in previous or subsequent components.

When attention is directed by exogenous cues, the interpretation becomes more complicated because the cue stimulus itself occurs in the vicinity of the target and causes a physiological response that is not likely to be independent from that of the target stimulus. Several researchers (Eimer, 1994; Hillyard, Luck & Mangun, 1994; Mangun, 1995; Miniussi, Wilding, Coull & Nobre, 1999) have tried to prevent cue-target interactions by using exogenous cues with cue-target intervals of at least 600 ms. They found changes only in the amplitudes but not in the latencies of the ERP components elicited by the target stimuli. However, the transient effects of exogenous cues have vanished by this time (Müller & Rabbitt, 1989; Nakayama & Mackeben, 1989). Other researchers have used shorter cue-target intervals and attempted to account for or subtract the effect of the cue to analyze the residual attentional effects on the target. Van der Lubbe and Woestenburg (1997) used a cue-

target interval as small as 100 ms and employed a trend analysis to account for the linear and nonlinear contributions to the evoked response from the cue. Bruin, Kenemans, Verbaten and van der Heijden (1998) used a cue-target interval of 140 ms and simply subtracted the ERPs for the cue alone from the cue plus target joint ERP. These studies did not show any changes in the latencies of the early ERP components, though van der Lubbe and Woestenburger suggested that changes in the N230 component could be interpreted as a latency shift.

There is really only one set of experiments that suggest that typical spatial cueing has any effect on the latencies of evoked potentials. Di Russo and Spinelli (1999) instructed observers to attend to a sustained peripheral cue that randomly changed color every 2.5–5.5 s. The cue was superimposed on either a gray background on one side of the visual field or on a grating contrast-reversing sinusoidally at 5–9 Hz on the other side. When grating was attended, the amplitude of the steady-state evoked potential (VEP) was increased relative to when the empty background was attended. The attended VEP had a latency of 135 ms relative to the oscillating stimulus, and the unattended VEP was delayed in phase by an additional 14 ms. In a control experiment where no attentional instructions were given, the presence or absence of the superimposed spot did not change the response to the grating, although curiously the latencies were shorter than in the previous experiments (93–94 ms compared to 135–149 ms). Other experiments investigating steady-state potentials evoked from stimuli oscillating at 8.6, 12, 20.8 or 27.8 Hz have found that endogenous attention increases the amplitude but does not consistently alter the phase (latency) of the response (Morgan, Hansen & Hillyard, 1996; Müller et al., 1998). The steady-state VEP data is difficult to interpret, because it is not necessarily clear what cognitive process the amplitude shifts correspond to.

Di Russo and Spinelli (1999) also tested a transient condition using the same cue as their steady-state experiment, but used a discrete oscillation of 0.5–0.8 Hz in order to investigate the evoked response to each stimulus change. When attention was directed to the grating rather than the background, they observed significant

increases in the amplitude of the P100 and N140 components, but not the N60 or P200 components. Attention also significantly shortened the peak latencies of the N60 and P100 components by 7 ms and 10 ms, respectively, and lengthened the latency of the P200 component by 5 ms.

In general, it seems that neither endogenous nor exogenous attention affects the latencies of early ERP components. Latency effects may be measured under specific conditions, but this does not appear to be a general phenomenon, and more investigation is needed.

## **II.6 Interactions between luminance and attention latencies**

Further evidence that endogenous attention does not affect early sensory processing comes from a comparison with the effects of luminance. Luminance modulates physiological latencies at an early stage of processing but attention apparently does not. However, both modulate the amplitudes of physiological responses, and it is possible to determine at what stage attention does affect latency by determining the first point of interaction between the attention and luminance effects. Johnannes, Munte, Heinze and Mangun (1995) showed that increasing the luminance of stimuli increased the amplitude of posterior N95 (80–110 ms), occipital P1 (110–140 ms) and parietal N1 (130–180 ms) components and reduced the latency of the occipital N1 component (135–220 ms). Attention (oriented by instructions to attend to one side and to detect an infrequently occurring stimulus) increased the amplitude of many occipital and parietal components. However, the first interaction between the luminance and spatial attention effects was not observed until the P3 (350–750 ms) component. There were no apparent differences in peak latencies of the attention difference waves for bright versus dim stimuli. Hawkins, Shafto and Richardson (1988) found that reaction time measures of attention interacted with stimulus luminance, with larger attention effects on reaction time for stimuli with lower

luminance, but these interactions may not reflect the earliest stages of perceptual processing.

Wijers, Land, Mulder and Mulder (1997) examined the effects of sustained spatial attention on ERPs induced by isoluminant and non-isoluminant stimuli. The average reaction time to and the peak latency of the P300 component evoked by isoluminant stimuli were about 80 ms slower. Isoluminance also increased the latency of the N80 component by 38 ms. The short latency of the effect suggests a subcortical origin of delay, and the additional later effect suggests that isoluminance also affects the decision process. No interactions were observed between the isoluminance condition and attention—the effects of attention were delayed in parallel in the isoluminant condition relative to the non-isoluminant condition. The difference between the evoked responses is similar in shape for the first few components, but delayed in time.

Hughes (1984) measured the dependence of reaction time on the strength of positional expectancy. In one experiment he showed that varying luminance over three orders of magnitude produced additive effects of luminance and attention on reaction time. A second experiment showed that the information value of the attentional cue affected the magnitude of the attentional effect. These effects were also additive with the luminance effects, suggesting that attention operates fairly late in the detection process (see Sternberg, 1998 for the logic of additive factors). Hawkins, Shafto and Richardson (1988) found that the attentional effects were larger for lower luminances if luminance was varied randomly within blocks instead of between blocks. In summary, while luminance changes cause measurable changes in early sensory processes that propagate through to the response, endogenous attention does not seem to cause similar changes.

## II.7 Explaining other phenomena with attentional acceleration

The prior entry hypothesis is widespread in the literature, used as an explanation for a number of phenomena, such as the line motion illusion and flash-lag effects (see below). Also, several researchers have used a temporal order judgment as an index of attentional allocation (e.g. Enloe, Illardi, Atchley, Cromwell & Sewell, 2001; Kakolewski, Crowson, Sewell & Cromwell, 1999; Stelmach, Campsall & Herdman, 1997). It is important not to draw conclusions about attention based on results of experiments that seem to show prior entry until the attentional contribution to the apparent prior entry effect has been conclusively demonstrated.

Hikosaka et al. (1993) hypothesized that differential perceptual latency across a spatial gradient of attention explains the line motion illusion, motion perceived in a static line that is preceded by an adjacent spatial cue. Though there are other explanations for the phenomenon (e.g. Downing & Treisman, 1997), several researchers have used the illusion to attempt to infer properties about attention (Steinman, Steinman & Lehmkuhle, 1995; Steinman, Steinman & Lehmkuhle, 1997; Steinman & Steinman, 1998), but they may have instead been reporting the properties of apparent motion.

Several researchers promote the prior entry hypothesis (e.g. Baldo & Klein, 1995; Purushothaman, Patel, Bedell & Ogmen, 1998), or non-attentional latency reductions (Bachmann & Pöder, 2001; Whitney, Cavanagh & Murakami, 2000; Whitney & Murakami, 1998), as an explanation for what has been termed the flash-lag effect by Nijhawan (1994), a variation of the complication experiment contained within the visual domain that was originally reported by Metzger (1932). In such experiments, a flashed object is observed to lag behind an object moving through either visual space or even feature space (Sheth, Nijhawan & Shimojo, 2000), even though the two objects are physically presented at the same time and location. Alternative hypotheses have also been formulated (e.g. Berry, Brivanlou, Jordan & Meister, 1999; Brenner & Smeets, 2000; Eagleman & Sejnowski, 2000; Kerzel, 2000;

Krekelberg, 2001; Krekelberg & Lappe, 1999; Lappe & Krekelberg, 1998; Nijhawan, 1994; Nijhawan, 1997; Sheth et al., 2000). Kirschfeld and Kammer (1999; 2000) used prior entry as part of an explanation for the similar Fröhlich effect (Fröhlich, 1923), in which a line first appears ahead of its actual position of origin.

## **II.8 Conclusions from background information**

A review of the literature suggests that exogenous cueing may cause an attentional acceleration effect, but that other forms of attentional orienting, such as trial-by-trial endogenous cues or sustained instructions, may not. Numerous experiments consistently do not reveal any effects of endogenous attention on the latency of physiological responses, and few physiological experiments have been performed with exogenous cues. Psychophysical experiments show that exogenous attentional cues produce robust apparent latency effects, while endogenous cues produce smaller or no effects. A large component of the psychophysical effects could be due to biases in the decision process rather than effects upon the perceptual latencies of the stimuli. In summary, the inconsistent results in the literature and the variety of interpretations suggest that the prior entry phenomenon is still ripe for experimentation.

## III Experiments

### III.1 Choice of task

If the attentional acceleration hypothesis is correct, then a stimulus to which attention is allocated should have a reduced latency of propagation through the brain compared to an otherwise identical but unattended stimulus. Two simultaneous stimuli, one attended and the other unattended, ought to appear successively, with the attended stimulus perceived first. It should also be possible to physically delay the onset of the attended stimulus relative to the unattended stimulus by an amount equal to the prior entry effect to recreate perceived simultaneity. The time difference at which maximal simultaneity is perceived should correspond to the point of maximal uncertainty in the TOJ, and if the two judgments systematically yield different measurements, then an early latency reduction due to attention cannot be the only cause of the apparent prior entry effects.

One of the predictions of the attentional acceleration hypothesis is that attention should also affect SJs. If the arrival time to the decision mechanism is reduced for attended versus unattended stimuli, then two physically simultaneous stimuli, one attended and one unattended, should be less likely to be judged simultaneous. If attention simply reduces the delay of attended stimuli, then it should also be possible to physically delay the attended stimulus to cancel the attentional effect. It is possible to compensate for luminance-dependent changes in latency in this way (Julesz & White, 1969). If the cause of the apparent prior entry effects is early, and if the decision mechanism for TOJs is free of any additional attentional effects, then an identical compensatory shift should be necessary to realign the stimuli for both TOJs and SJs.

Only two known studies have investigated SJs and attention. In Stelmach and Herdman's (1991) Experiment 5, observers adjusted the delay between two stimuli, one occurring at an exogenously cued location. To achieve the appearance of



maximal simultaneity, the unattended stimulus had to lead the attended stimulus by about 20 ms, smaller than the 40 ms effects reported in their TOJ experiments.

Carver and Brown (1997) investigated the effects of exogenously cueing either both or neither of stimuli in SJs. They found that pairs of stimuli in exogenously cued locations were more likely to be judged as simultaneous across a range of asynchronies between the stimuli than were pairs of stimuli in uncued locations. They interpret this as indicating that attention narrows the interval within which two stimuli are judged as simultaneous.

Although TOJs and SJs may be made concurrently, either separately as in Allan (1975), or combined as in Ulrich's (1987) ternary response tasks, I will analyze them independently. Ternary response tasks allow the observer the choice of responding either with the order of the stimuli, or a third option that the stimuli were simultaneous. In such a task, the level of certainty required for a "simultaneous" rather than an order response might vary among observers or conditions. For example, in Stelmach and Herdman's (1991) Experiment 4, the observers rarely utilized the "simultaneous" response when attention was directed either to the left or right. It is also possible that SJs and TOJs do not employ the same underlying mechanism. For example, the appearance of the attentional cues might interfere with the simultaneity mechanism but leave the order mechanism intact.

Judgments of simultaneity and temporal order are closely related theoretically. Hirsh and Sherrick (1961) suggested that to identify order, successiveness must first be perceived. According to Ulrich (1987), simultaneous responses should correspond to the point of greatest uncertainty in the judgment of temporal order. Allan (1975) investigated the relationship between the two types of judgments and assumed that the perception of successiveness was sufficient for the perception of order. In her experiments, observers made either a SJ, a TOJ, or both judgments in succession for the offsets of a light and a tone. She found that an attention-switching model best accounted for the data, and that under many other models, the SJ and TOJ data could not be derived from a single underlying distribution of latency differences between

the stimuli. A decision mechanism that included a differential response bias between the judgments might account for the discrepancy.

### **III.2 Properties of the visual system affecting temporal order judgments**

In designing experiments to test the order and simultaneity of stimuli, it is important to control for intrinsic properties of the visual system that could influence the results.

#### **III.2.1 Eccentricity**

The visual system can detect discrepancies in synchrony and temporal order within a few milliseconds, though results depend on retinal position and viewing conditions. Rutschmann (1966) reported that 500 ms flashes in the fovea needed to be delayed 40 to 50 ms relative to those in the periphery to maximize the uncertainty of temporal order judgment. For this reason, stimuli will be displayed at a constant eccentricity in these experiments.

#### **III.2.2 Apparent motion**

Care must be taken to prevent observers from employing motion mechanisms to determine temporal order. Westheimer and McKee (1977) report that by perceiving the direction of apparent motion, observers can correctly identify the temporal order of two adjacent stimuli when the delay between them is as short as 3 ms for an optimal spatial configuration and binocular viewing. Westheimer (1983) found that these optimal thresholds did not vary greatly with retinal eccentricity, though the dependence on the spatial configuration of the stimuli did vary near the fovea.

Similar to Exner's (1875) findings, Allik and Kreegipuu (1998) showed the precision of detecting the temporal order of stimuli was much better when the stimuli were spatially located to permit a perception of apparent motion between them. For

the disjoint stimuli (separated by  $10^\circ$ ), the point of subjective equality shifted 32.3 ms over a 16-fold increase in luminance, while for adjacent stimuli (separated by  $0.04^\circ$ ) exhibiting apparent motion, the shift was only 3.8 ms. Similarly, Carver (1998) showed the threshold for detecting motion was smaller than that for detecting the successiveness of two stimuli. The specialized motion system does not seem to be sensitive to luminance-provoked latency differences and seems to have access to high time resolution information not accessible to the presumably conscious perceptual subsystems used in determining stimuli onsets, temporal orders or locations. Allik and Kreegipuu also showed that, especially at low luminances, reaction time was decreased to a stimulus presented subsequent to an adjacent stimulus after a delay optimal for apparent motion (60 ms) compared to after a non-optimal delay (600 ms). However, this cannot be differentiated from an attentional account, since at this timescale, the first stimulus acts as an effective exogenous attentional cue.

### **III.3 Experiment 1. Exogenous cues.**

#### **III.3.1 Introduction**

The first experiment employs exogenous cues to orient attention. Based on previous results, these are expected to cause the largest shifts on the PSS. The purpose of this first experiment is to replicate previous results using a new experimental paradigm, and to compare the attentional effects between TOJs and SJs.

#### **III.3.2 Methods**

##### ***III.3.2.1 Materials***

All experiments were performed using a Macintosh G3 computer (Apple Computer, Inc., Cupertino, CA) running a program to present the stimuli and collect the data

written in the Matlab computer language (The Math Works, Inc., Natick, MA) with Psychophysics Toolbox routines (Brainard, 1997; Pelli, 1997). The stimuli were displayed on a ViewSonic P817 21-inch monitor (ViewSonic, Inc., Walnut, CA) driven at approximately 160 Hz by a MP 850 video card (Village Tronic Computer, Sarstedt, Germany). The resolution of the presentation timing for the stimuli were thus quantized by the monitor refresh time of 6.25 ms such that the reported stimuli timings, though consistent, might differ from the actual timing by as much as half this value. The screen background color was a uniform gray, 12.5 cd/m<sup>2</sup>, with CIE (Commission International d'Eclairage) color coordinates  $x = 0.289$  and  $y = 0.314$ , and the experiment took place in a room dimly illuminated by indirect lighting. Responses were collected on the computer keyboard, with an estimated accuracy of  $\pm 8$  ms.

### *III.3.2.2 Observers*

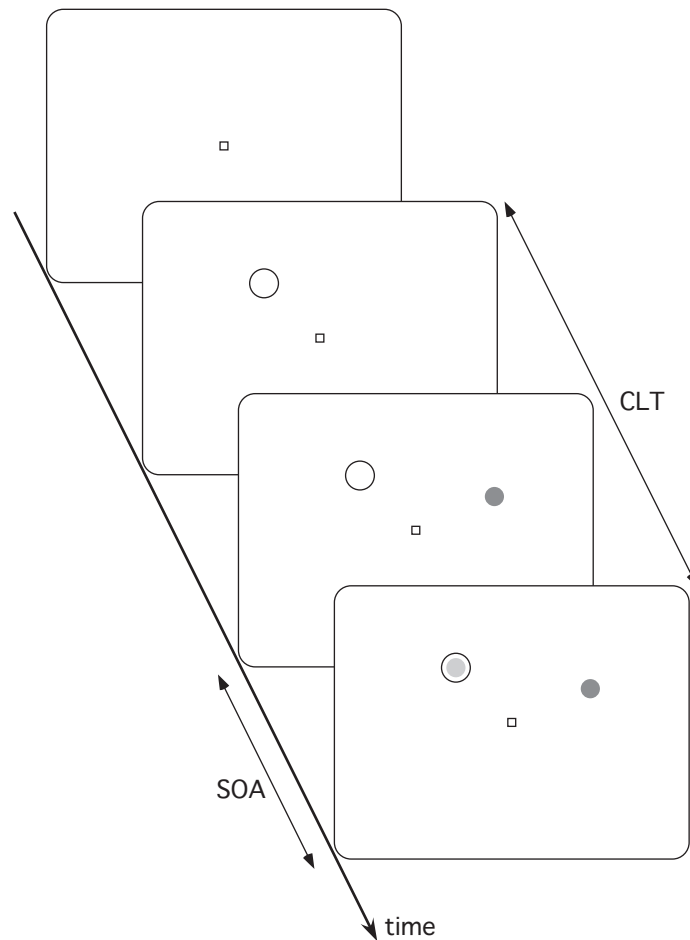
Nine graduate students at the University of Rochester, aged 23–33 years, five female, participated in the experiment. One observer reported being left-handed. All reported normal or corrected to normal vision, and all were naïve to the purpose of the experiment, although most were experienced psychophysical observers. Six observers (NV, LS, SBS, JT, SC, MM) had previously participated in Experiments 2–4, two observers (MB, MC) had previously participated in Experiments 2–3, and one observer (VI) had previously participated in Experiment 4. The observers were each paid a total of \$20 for both blocks of the experiment.

### *III.3.2.3 Stimuli*

Two 0.3° circular targets were presented at pseudorandom positions, constrained to be separated by at least 7° and both located within an annulus centered at the fixation point, at a radius uniformly ranging from 6.5–7.5°. The colors of the two targets were

red ( $x = 0.623$ ,  $y = 0.340$ ) and green ( $x = 0.292$ ,  $y = 0.601$ ), equated for brightness at a luminance level of approximately  $16 \text{ cd/m}^2$  by minimizing heterochromatic flicker (see e.g. Tansley & Boynton, 1978) through the following procedure. While the observer fixated on a small central fixation point, two targets were presented in the periphery at typical positions on the screen as used throughout the experiments. The colors of the targets were alternated at 15 Hz between red and green, and the observers varied the ratio of the intensities of the two colors (one always being maximally bright) through key presses until they reported that the flickering of the targets was minimized.

One cue, a thin  $0.5^\circ$  white ( $84.2 \text{ cd/m}^2$ ,  $x = 0.286$ ,  $y = 0.309$ ) ring, appeared concentric to one of the targets. The other cue remained uncued and could appear before or after the cued target, as described above. The cue either occurred simultaneously with its target, or preceded it by a cue lead time (CLT) of 40, 75, 125, 200, 500 or 1000 ms. The stimulus time course is illustrated in Figure 3.



**Figure 3.** An example stimulus frame time sequence for Experiment 1. The cue lead time (CLT) between the cue and the cued target and stimulus onset asynchrony (SOA) between the cued and uncued targets are shown. The illustrated SOA is negative because the uncued target led the cued target. The figure is not to scale—for clarity, the sizes of the targets and cue have been exaggerated relative to their distances.

### *III.3.2.4 Procedure*

The experiment consisted of two parts. In separate blocks on separate days, the observers judged either the order or simultaneity of stimuli. The observers were divided into two groups, one making the SJ on the first day, and the other making the TOJ. For the SJ, the observers were instructed to press one key if the two targets

appeared simultaneously (“at the same time”), and to press another key if the targets appeared successively (“at different times, or one before the other”). For the TOJ, the observers were instructed to press a key indicating the color of the target that appeared first. In both cases the computer would proceed to the next stimulus presentation only when the observers pressed one of the two response keys.

Observers were instructed to make their best guess when uncertain, and that although their responses were not timed, that a prolonged decision would not be helpful, and to respond as quickly as possible while maintaining accuracy. Observers were seated with their eyes 55 cm from the fixation point and were instructed to maintain fixation. The observers’ heads were not restrained, and their eye movements were not monitored.

The method of constant stimuli was used to sample the psychometric function, and the order and interval between the two targets varied. The two targets appeared either simultaneously, or with an SOA of 25, 50, 75 or 100 ms. The cues and targets remained visible until the observers indicated a response. The fifty-four combinations of cuing lead times and interstimulus intervals were each repeated twenty times with all conditions randomly interleaved. Each experimental session lasted approximately one hour, during which the observers were automatically allowed to rest and break fixation after every fifty stimulus presentations and to resume the experiment when ready. Text on the computer monitor reminded the observers to maintain fixation before resuming. The observers were given no feedback about their progress other than being informed when they were 25, 50, 75 and 90% complete.

### III.3.3 Results

Although the observers were instructed that their responses were not timed, their reaction time (beginning from the onset of the second target stimulus) was monitored to allow the exclusion of abnormally delayed decisions. To determine outlier points,

the skew (as defined by the quotient of the third central moment and the cube of the standard deviation) of the distribution of reaction times was calculated for each observer, each CLT, and each SOA. If the skew was greater than 1.2, the trial with the maximum reaction time was discarded, and the procedure was iterated until the skew of the distribution with the remaining trials was less than 1.2. For each observer, an average of 5.9% of the trials in both the SJ block and TOJ block were excluded from further analysis in this manner. Reaction time was not a planned dependent variable of interest and is not analyzed further in this or subsequent experiments.

For a superficial description of the data (a mechanistic analysis is used in Part III), the average PSS across all observers for each SJ and TOJ, and for each CLT, were estimated from the data as the weighted average of the maximum likelihood estimates of the means  $\mu$  obtained from fits to scaled normal,

$$\phi(\Delta t, \mu, \sigma, h) = h \exp\left(-(\Delta t + \mu)^2 / 2\sigma^2\right), \text{ and cumulative normal, } \Phi(\Delta t, \mu, \sigma) \text{ (see}$$

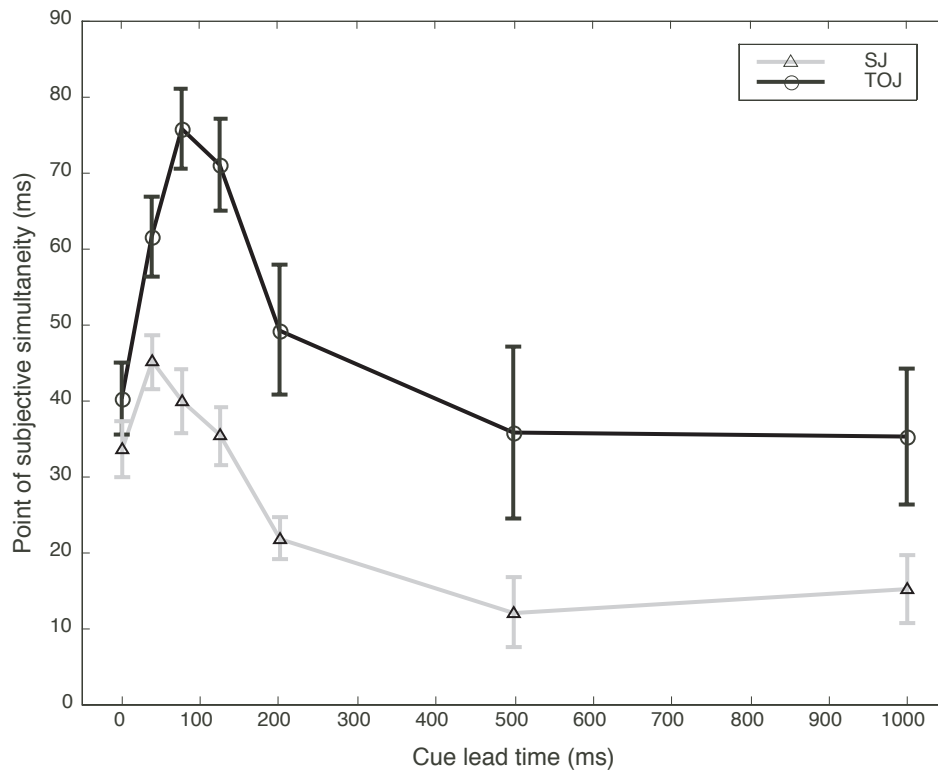
Equation 4 in Part III) distribution, for the SJ and TOJ respectively. The  $\Delta t$  values are the SOAs between the two stimuli. The fitting procedure is described in detail in Part IV, and yields estimates and errors of the function parameters. Of the sixty-three combinations of observer and CLT condition, four had to be eliminated from further analysis because of the failure of the optimization procedure to estimate the error parameters. The weighted average and variance of each parameters from  $N$  observers was calculated as



$$\begin{aligned}
\bar{x} &= \frac{\sum_{k=1}^N w_k x_k}{\sum_{k=1}^N w_k} \quad \text{and} \\
s^2 &= \frac{\left( \sum_{k=1}^N w_k \right) \sum_{k=1}^N w_k (x_k - \bar{x})^2}{\left( \sum_{k=1}^N w_k \right)^2 - \sum_{k=1}^N w_k^2},
\end{aligned} \tag{1}$$

where  $x_k$  is the maximum likelihood estimator of the parameter for the observer  $k$ ,  $w_k = \varepsilon_k^{-2}$ , and  $\varepsilon_k$  is the standard deviation of the parameter estimate determined from the fitting procedure. Statistics can be performed on the weighted average estimates by noting that  $\frac{(\bar{x} - \mu)}{s/\sqrt{N}} \sim t_{N-1}$ , where  $t_{N-1}$  is the  $t$  distribution with  $N-1$  degrees of freedom.

The results are plotted in Figure 4 and listed in Table 1. As can be seen, the PSS is significantly different from zero for every CLT for both the SJ and TOJ. Further, the PSS for the SJ and TOJ differed significantly, or nearly so, from each other at each CLT other than 0 ms, when the cue was simultaneous with its target. The PSSs determined from the TOJ were consistently larger than those determined from the SJ. The PSSs increase rapidly to a maximum at a CLT less than 100 ms and then diminish to a sustained level near the CLT of 200 ms.



**Figure 4.** Shifts in the points of subjective simultaneity (PSS) for the simultaneity (SJ) and order (TOJ) judgments caused by the exogenous cues in Experiment 1. The PSSs are significantly different than zero for all cue lead times and differed significantly between the judgments for all cue lead times between 40–200 ms. Error bars indicate the standard error of the weighted means.

**Table 1.** Mean shifts in the points of subjective simultaneity (PSS) and their significance levels produced by the exogenous cues in Experiment 1.

Cue lead time (ms)	0	40	75	125	200	500	1000
PSS <sub>SJ</sub> (ms)	33.4	45.0	39.7	35.2	21.6	12.0	14.9
$p(\text{PSS}_{\text{SJ}} = 0)$	<.001	<.001	<.001	<.001	<.001	.03	.008
PSS <sub>TOJ</sub> (ms)	40.2	61.5	75.7	71.0	49.3	35.8	35.3
$p(\text{PSS}_{\text{TOJ}} = 0)$	<.001	<.001	<.001	<.001	<.001	.01	.004
$p(\text{PSS}_{\text{SJ}} = \text{PSS}_{\text{TOJ}})$	n.s.	.02	<.001	<.001	.007	.06	.06

### III.3.4 Discussion

The presence of the exogenous cue significantly affects both the SJ and TOJ, with dynamics typical of an exogenous attentional process. However, the discrepancy between the two judgments does not support a single underlying cause of the shift in the PSS. The TOJ may be observer to response biases that are not present for the SJ, and it is also possible that the SJ and TOJ decision mechanisms operate upon different latency distributions derived from different measurements of the target stimuli onsets.

## **III.4 Experiment 2. Endogenous cues.**

### III.4.1 Introduction

One possibility is that the transient activity caused by the exogenous cue at the target location could be misidentified with or incorporated into the timing of the cued target. Some sensory interactions may occur between the cue and the target because they both stimulate similar or identical receptors and neurons in the ascending visual pathways. The cue might induce either excitatory or refractory states that could influence the processing of subsequent stimuli. To reduce this possibility, attention was directed in this second experiment with an endogenous cue—a central arrow—located remotely from the targets. Jonides (1981) observed that an exogenous cue drew attention automatically, whether informative or not, while an endogenous cue required a deliberate shift of attention and only controlled the deployment of attention when informative. The central arrow in this task provides strictly informative location information about one of the two targets, to maximize its attention directing capabilities (Hughes, 1984), though the cue itself was irrelevant for the task. Previous research (see above) has yielded mixed results whether endogenous cues cause a prior entry effect.

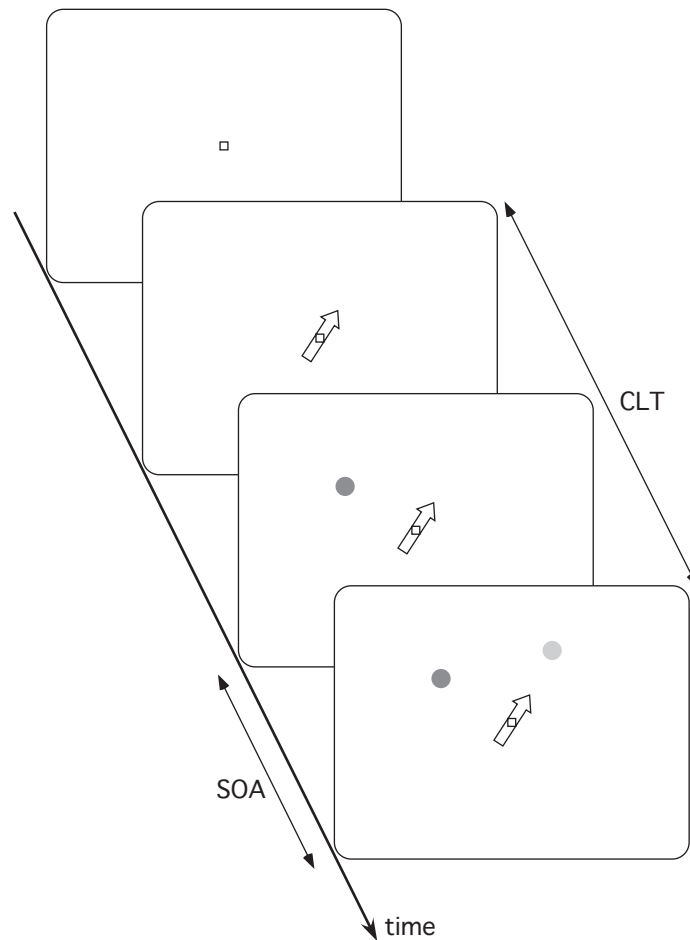
### III.4.2 Methods

#### *III.4.2.1 Observers*

Nine graduate students at the University of Rochester, aged 21–32 years, six female, participated in the experiment. One observer reported being left-handed. All reported normal or corrected to normal vision, and all were naïve to the purpose of the experiment, although most were experienced psychophysical observers. All observers had previously participated in Experiment 3. The observers were each paid a total of \$20 for both blocks of the experiment.

#### *III.4.2.2 Procedure*

The method was similar to that for Experiment 1 with the following exceptions. A central arrow cue was used instead of cues at the target location. A white (84.2 cd/m<sup>2</sup>,  $x = 0.286$ ,  $y = 0.309$ ) arrow, whose tip extended 1.5° from the fixation point, appeared at the fixation point in a random orientation. After a delay of 0, 100, 300, 600, 1000 or 1500 ms, the cued target appeared at a location whose angle relative to the fixation point matched the orientation of the arrow. As in Experiment 1, the uncued target appeared either simultaneously with the cued target, or preceded or followed it by an SOA of 25, 50, 75 or 100 ms. The stimulus time course is illustrated in Figure 5.

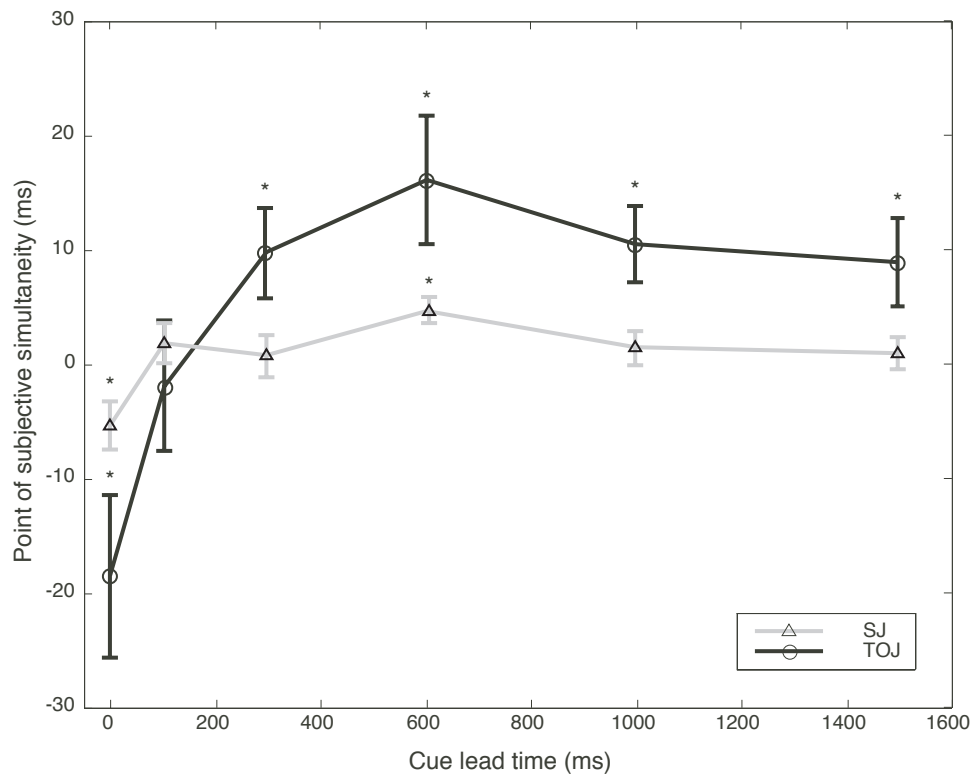


**Figure 5.** An example stimulus frame time sequence for Experiment 2. The cue lead time (CLT) between the cue and the cued target and stimulus onset asynchrony (SOA) between the cued and uncued targets are shown. The illustrated SOA is negative because the uncued target led the cued target. The figure is not to scale—for clarity, the sizes of the targets and arrow have been exaggerated relative to their distances.

### III.4.3 Results

An average of 5.8% of the trials in the SJ block and 5.1% in the TOJ block for each observer were excluded from further analysis due to delayed reaction times, following the procedure described in Experiment 1.

The PSSs are calculated as in Experiment 1 and appear in Figure 6 and Table 2. The PSSs for the TOJ differed significantly from zero at the CLTs of 300 ms or longer, and the PSSs for the TOJ and SJ differed significantly, or nearly so, from each other at these CLTs as well. Of the data taken at positive CLTs, only the PSS for the CLT of 600 ms was significantly different from zero for the SJ, and even this effect was small (4.7 ms). When the cue and target were simultaneous (CLT equals zero), the PSSs for both the TOJ and SJ were negative and differed significantly from zero.



**Figure 6.** Shifts in the points of subjective simultaneity (PSS) for the simultaneity (SJ) and order (TOJ) judgments caused by the endogenous cues in Experiment 2. The asterisks indicate the PSSs that are significantly different from zero. The two judgments differed at least marginally significantly at the cue lead times of 300 ms and longer. Error bars indicate the standard error of the weighted means.

**Table 2.** Mean shifts in the points of subjective simultaneity (PSS) and their significance levels produced by the endogenous cues in Experiment 2.

Cue lead time (ms)	0	100	300	600	1000	1500
PSS <sub>SJ</sub> (ms)	-5.2	2.0	0.8	4.7	1.5	1.1
$p(\text{PSS}_{\text{SJ}} = 0)$	.04	n.s.	n.s.	.002	n.s.	n.s.
PSS <sub>TOJ</sub> (ms)	-18.5	-1.9	9.8	16.1	10.6	8.9
$p(\text{PSS}_{\text{TOJ}} = 0)$	.03	n.s.	.04	.02	.01	.04
$p(\text{PSS}_{\text{SJ}} = \text{PSS}_{\text{TOJ}})$	.09	n.s.	.06	.06	.02	.07

#### III.4.4 Discussion

The central arrow cue generally only has significant effects for the TOJ and not the SJ. The TOJ effects only become significant after a CLT of 300 ms, consistent with the effects of endogenous attention. As with Experiment 1, the results differed between the SJ and TOJ. It is possible that the TOJ effects are due only to response biases that are absent from the SJ. When the arrow was presented simultaneously with the cued target, it seemed to have an inhibitory effect such that the cued target was seen as occurring after the uncued target. The cause of this effect is unclear.

### III.5 Experiment 3. Gaze-directed cues.

#### III.5.1 Introduction

One criticism of Experiment 2 could be that the central arrow cue did not efficiently orient attention towards the cue it pointed to. Unlike exogenous cues, endogenous cues are not automatic (Jonides, 1981). In an attempt to direct attention more efficiently but still refrain from introducing transient activity at the target location, I employed what are known as gaze-directed or social-reflexive cues.

Recently (see Langton, Watt & Bruce, 2000, for a review), researchers have found that reflexive covert shifts of visual attention to peripheral locations can be triggered by uninformative eye-gaze direction presented at fixation, both in cartoon faces (Friesen & Kingstone, 1998; Kingstone, Friesen & Gazzaniga, 2000) and pictures of real faces (Driver et al., 1999; Hietanen, 1999; Langton & Bruce, 1999; Langton & Bruce, 2000). Reaction time is facilitated when a target appears at the cued location where the eyes were looking. This facilitation effect occurred as early as 105 ms, disappeared by 1005 ms, and was not accompanied by any reaction time delay at uncued locations relative to a neutral condition (Friesen & Kingstone, 1998). Driver (1999) found gaze direction caused faster discrimination of peripheral letters, even when the letters were four times as likely to occur in an opposite location. When the gaze cue countered the target location probability in this way, the gaze cue significantly facilitated discrimination time only at 300 ms, not earlier or later, suggesting that gaze-directed shifts of attention were only automatic for a limited duration. Interestingly, reflexive gaze-directed attention also seems to reduce the latency of the P1 and N1 occipito-parietal ERP components (Schuller & Rossion, 2001), perhaps providing physiological evidence for the attentional acceleration hypothesis.

### III.5.2 Methods

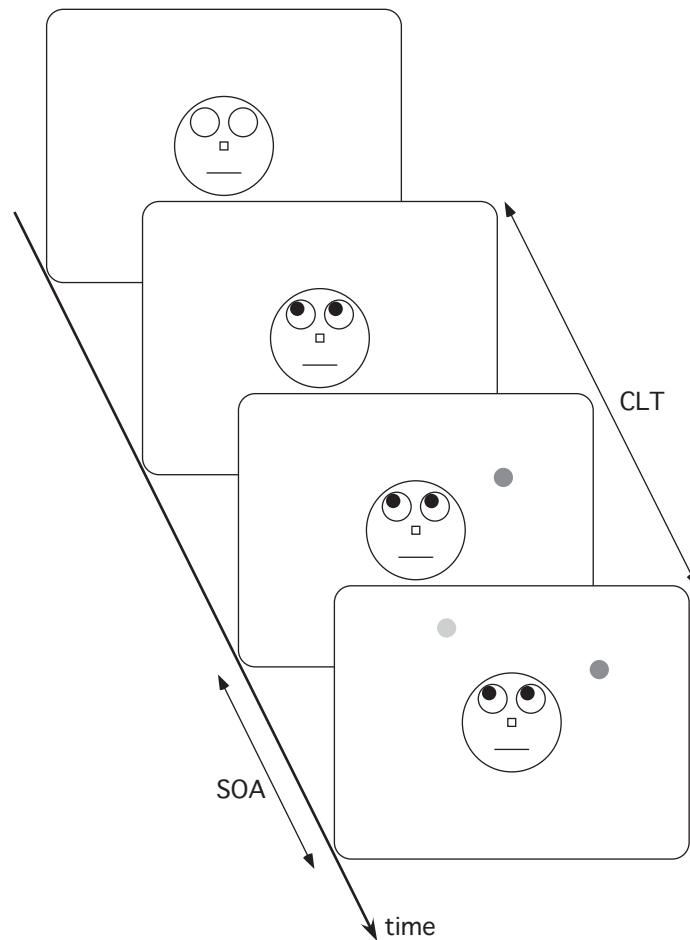
#### *III.5.2.1 Observers*

Ten graduate students at the University of Rochester, aged 21–32 years, seven female, participated in the experiment. One observer reported being left-handed. All reported normal or corrected to normal vision, and all were naïve to the purpose of the experiment, although most were experienced psychophysical observers. The observers were each paid a total of \$20 for both blocks of the experiment.



### *III.5.2.2 Procedure*

The method was similar to that for Experiment 2 with the following exceptions. Instead of a central arrow, a cartoon face,  $4^\circ$  in diameter, was drawn to the screen with white eyes and no pupils. After 500–1000 ms, the two pupils,  $0.3^\circ$  in diameter appeared such that the gaze of the face was directed at a region of the screen located within an annulus  $6.5\text{--}7.5^\circ$  from the fixation point, which was also the nose of the face. After a CLT of 0, 100, 300, 600, 1000 or 1500 ms, the cued target appeared at the location to which the face's gaze was directed. As in the previous two experiments, the uncued target appeared within 100 ms before or after the cued target. The stimulus time course is illustrated in Figure 7.

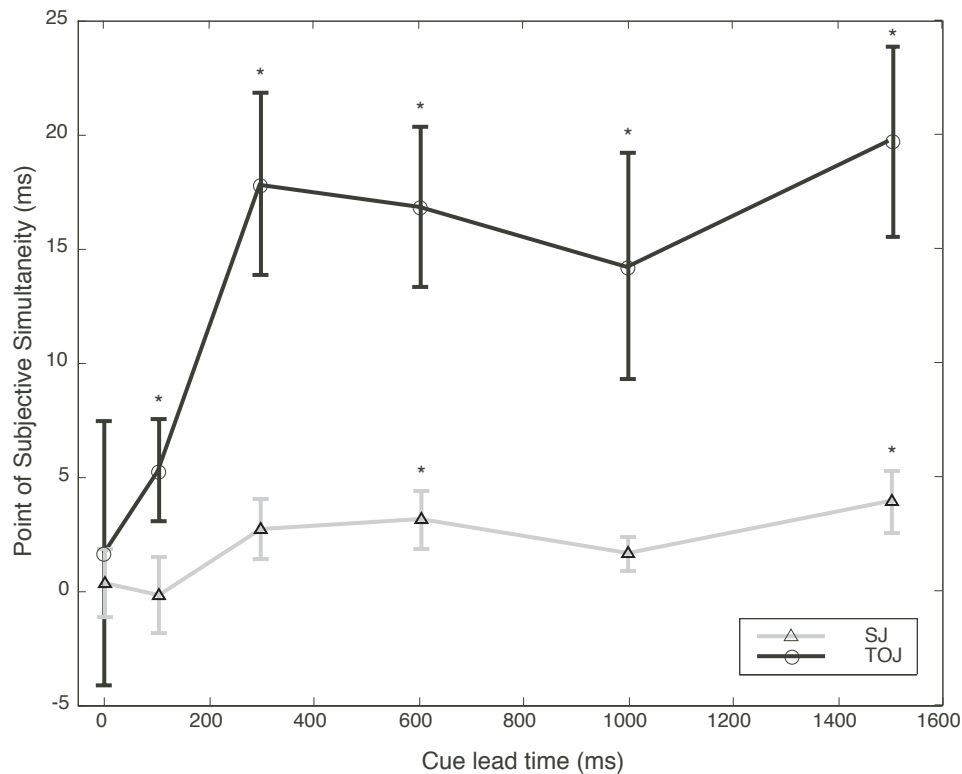


**Figure 7.** An example stimulus frame time sequence for Experiment 3. The cue lead time (CLT) between the cue and the cued target and stimulus onset asynchrony (SOA) between the cued and uncued targets are shown. The illustrated SOA is negative because the uncued target led the cued target. The figure is not to scale—for clarity, the sizes of the targets and the face have been exaggerated relative to their distances.

### III.5.3 Results

An average of 6.2% of the trials in the SJ block and 5.5% in the TOJ block for each observer were excluded from further analysis due to delayed reaction times, following the procedure described in Experiment 1.

The PSSs were calculated as in Experiment 1. The results were similar to those of Experiment 2 and appear in Figure 8 and Table 3. The PSS for the TOJ was significant at a CLT of 100 ms and increased to a sustained level by 300 ms, differing significantly from zero and from the SJ at these later points. The PSSs determined by the SJ were very small, but also significant or nearly so at CLTs of 300 ms or later.



**Figure 8.** Shifts in the points of subjective simultaneity (PSS) for the simultaneity (SJ) and order (TOJ) judgments caused by the gaze cues in Experiment 3. The asterisks indicate the PSSs that are significantly different from zero. The two judgments differed significantly at the cue lead times of 300 ms and longer. Error bars indicate the standard error of the weighted means.

**Table 3.** Mean shifts in the points of subjective simultaneity (PSS) and their significance levels produced by the gaze-directed cues in Experiment 3.

Cue lead time (ms)	0	100	300	600	1000	1500
PSS <sub>SJ</sub> (ms)	0.2	-0.3	2.6	2.9	1.5	3.7
$p(\text{PSS}_{\text{SJ}} = 0)$	n.s.	n.s.	.08	.05	.08	.02
PSS <sub>TOJ</sub> (ms)	1.6	5.3	17.8	16.8	14.2	19.7
$p(\text{PSS}_{\text{TOJ}} = 0)$	n.s.	.04	.001	<.001	.02	<.001
$p(\text{PSS}_{\text{SJ}} = \text{PSS}_{\text{TOJ}})$	n.s.	.06	.002	.002	.02	.002

### III.5.4 Discussion

The dynamics of the results for the TOJ are typical of gaze-directed attention, beginning slightly earlier than would the effects of endogenous attention. The similarity of the results of Experiments 2 and 3 suggest that it was not the lack of efficiency of the endogenous cue that caused the null results for the SJ, but the lack of an abrupt onset near the target location. This hypothesis is further tested in Experiments 4 and 5.

## III.6 Experiment 4. Isoluminant exogenous cues.

### III.6.1 Introduction

The purpose of this experiment is to further investigate the differing pattern of results observed between exogenous and non-exogenous cues. The exogenous cues produce sensory activity at the location of the target, and some aspect of this activity could be affecting judgments made about the targets (see Nakayama & Mackeben, 1989, for a discussion of transients). In this experiment the abrupt luminance change of the cues

is replaced by an isoluminant change in order to try to reduce or change the nature of the sensory transients.

### III.6.2 Methods

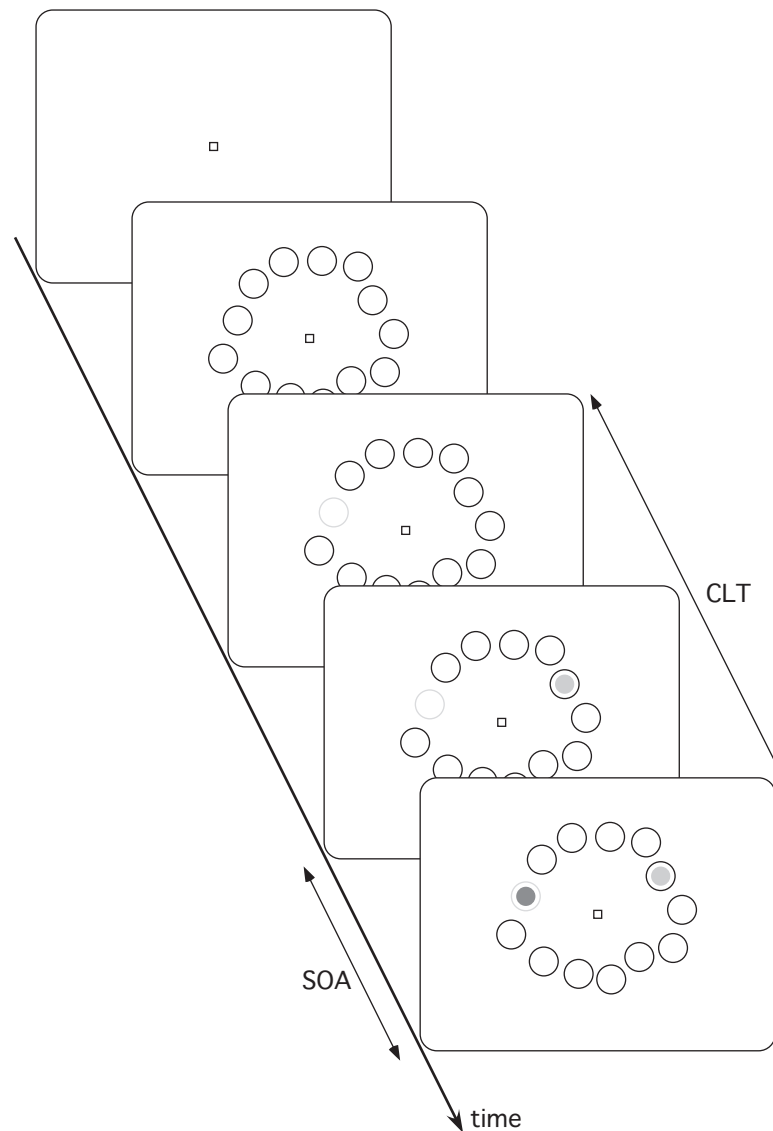
#### *III.6.2.1 Observers*

Eight graduate students at the University of Rochester, aged 23–33 years, four female, participated in the experiment. One observer reported being left-handed. All reported normal or corrected to normal vision, and all were naïve to the purpose of the experiment, although most were experienced psychophysical observers. Six observers (NV, LS, SBS, JT, SC, MM) had previously participated in Experiments 2–3, and one observer (MC) had previously participated in Experiments 1–3. The observers were each paid a total of \$20 for both blocks of the experiment.

#### *III.6.2.2 Procedure*

The method was similar to that for Experiment 1 with the following changes. Thirteen thin  $0.5^\circ$  cyan ( $x = 0.210$ ,  $y = 0.299$ ) rings appeared within  $6.5\text{--}7.5^\circ$  of the fixation point, each spaced a distance at least  $2.5^\circ$  from each other. After 750–1000 ms, one of these rings would change colors to an isoluminant yellow ( $x = 0.403$ ,  $y = 0.515$ ). The cyan and yellow colors were equated in brightness at a luminance level of approximately  $70 \text{ cd/m}^2$  for each observer by placing the cue rings at eccentricities used in the experiment and using the flicker photometry procedure described in Experiment 1. After a CLT of 0, 40, 75, 125, 200, 500 or 1000 ms, the cued target would then appear concentric to one of the rings that had changed color. The uncued target appeared within one of the rings that did not change colors, within 100 ms before or after the cued target. In order to contain the experiment with about an hour given the extra stimulus delay, each combination of CLT and SOA was repeated only

seventeen times, compared to twenty in the other experiments. The stimulus time course is illustrated in Figure 9.

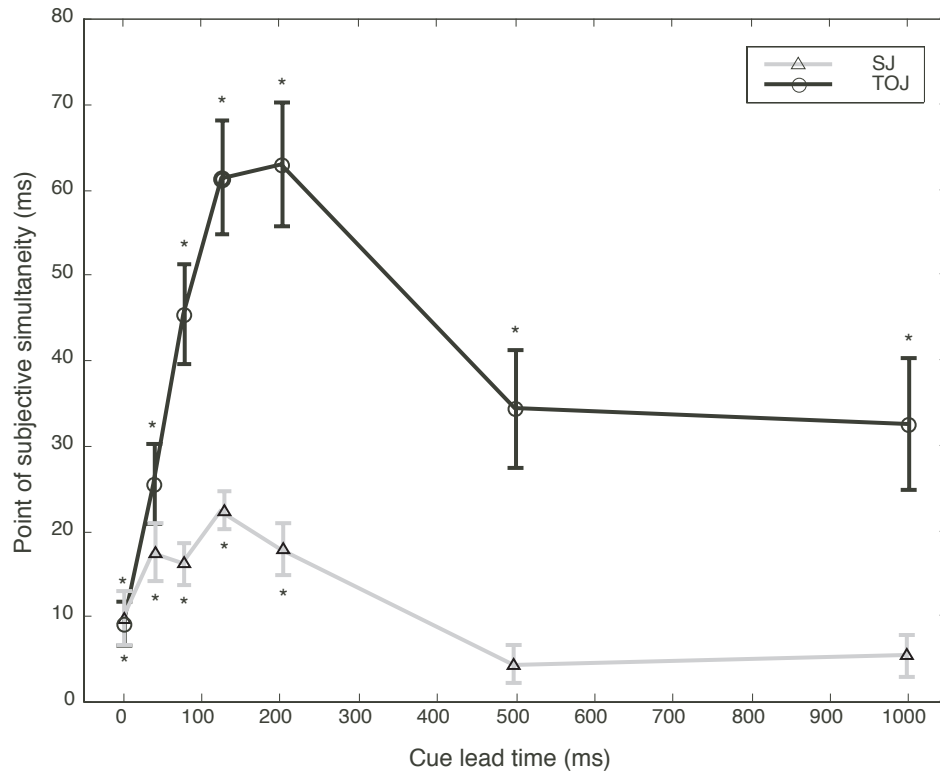


**Figure 9.** An example stimulus frame time sequence for Experiment 4. The cue lead time (CLT) between the cue and the cued target and stimulus onset asynchrony (SOA) between the cued and uncued targets are shown. The illustrated SOA is negative because the uncued target led the cued target. The figure is not to scale—for clarity, the sizes of the targets and cues have been exaggerated relative to their distances.

### III.6.3 Results

An average of 5.8% of the trials for the SJ block and 4.9% for the TOJ block for each observer were excluded from further analysis due to delayed reaction times, following the procedure described in Experiment 1.

The PSSs were calculated as in Experiment 1 and appear in Figure 10 and Table 4. The effects are similar to those in Experiment 1, with a somewhat delayed time course. The PSS for the TOJ peaked between 100–200 ms and decreased to a sustained level by 500 ms. The PSS for the SJ peaked somewhat earlier, around 125 ms. The PSSs determined by the SJ and TOJ were all significantly or nearly significantly different from zero, and the judgments differed significantly from each other at all but the shortest two CLTs.



**Figure 10.** Shifts in the points of subjective simultaneity (PSS) for the simultaneity (SJ) and order (TOJ) judgments caused by the isoluminant exogenous cues in Experiment 4. The asterisks indicate the PSSs that are significantly different from zero. The two judgments differed significantly from each other at all but the shortest two cue lead times. Error bars indicate the standard error of the weighted means.

**Table 4.** Mean shifts in the points of subjective simultaneity (PSS) and their significance levels caused by the isoluminant exogenous cues in Experiment 4.

Cue lead time (ms)	0	40	75	125	200	500	1000
PSS <sub>SJ</sub> (ms)	9.9	17.6	16.3	22.5	18.0	4.5	5.6
$p(\text{PSS}_{\text{SJ}} = 0)$	.01	<.001	<.001	<.001	<.001	.08	.05
PSS <sub>TOJ</sub> (ms)	9.1	25.5	45.5	61.4	62.9	34.3	32.4
$p(\text{PSS}_{\text{TOJ}} = 0)$	.008	<.001	<.001	<.001	<.001	.001	.003
$p(\text{PSS}_{\text{SJ}} = \text{PSS}_{\text{TOJ}})$	n.s.	n.s.	<.001	<.001	<.001	.001	.005



### III.6.4 Discussion

The delay of the cue effect relative to Experiment 1 could be due to the relative transmission delay between the color-defined and luminance-defined stimuli (see e.g. Wijers et al., 1997). Other than this delay, the isoluminant exogenous cues had similar sustained and transient effect components. The time course is typical of an exogenous attentional process, and again there was a discrepancy between the PSSs determined from the SJ and TOJ. The additional effect on the TOJ could be due to response biases present in the TOJ but not SJ. That the effects of the cues still occur when luminance-defined transients are decreased or eliminated suggests that luminance transients are not the cause of the exogenous cue effects.

## **III.7 Experiment 5. Multiple exogenous cues.<sup>†</sup>**

### III.7.1 Introduction

The purpose of this experiment is to try to determine whether attention directed by exogenous cues causes the observed effects on order and simultaneity judgments, or whether the effects are due to some other properties of the cues, such as sensory facilitation or perhaps confusion between the cue onset and the target onset. For example, Fendrich and Corballis (2001) suggested, based on their version of the visual-auditory complication experiment, that the perceived timing of two temporally proximal stimuli is shifted towards temporal convergence. In order to rule out such effects between the exogenous visual cue and target, and to investigate whether attention or stimulus interaction is the cause of the observed prior entry effects, it is desirable to test a case in which an abruptly onset cue is still present at the target location, but the attention drawn by the cue is minimized.

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<sup>†</sup> The results of this experiment were first presented at the May, 2001 meeting of the Visual Sciences Society in Sarasota, Florida (Schneider & Bavelier, 2001).

Yantis and Johnson (1990) found that in displays containing multiple stimuli with abrupt onsets, only a limited number, approximately four, of them could be assigned a high priority. Therefore, by presenting a larger number of cues, it should be possible to reduce the probability that attention is directed to any individual cued location. In this way, it should be possible to reduce the attentional effects of exogenous cues while still retaining their abrupt onsets in the proximity of the target. If the effects of the cues on the SJ and TOJ are due to their sensory facilitation rather than their capacity to direct attention, one would predict that increasing the number of cues would not have a significant affect on SJs or TOJs.

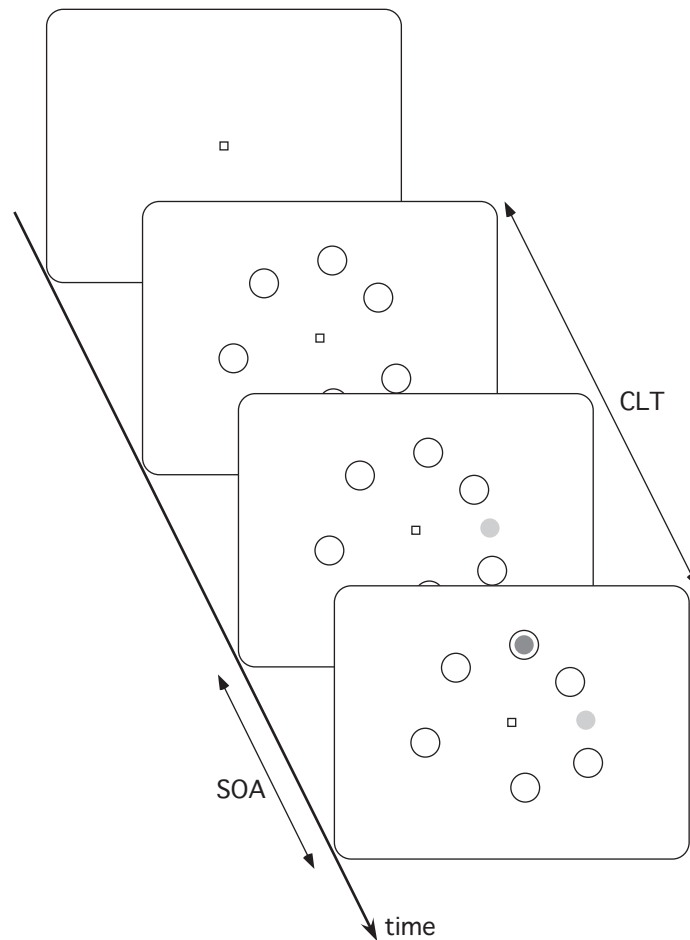
### III.7.2 Methods

#### *III.7.2.1 Observers*

Twelve undergraduate students at the University of Rochester, eight female, aged 13–31 years, participated in the experiment. One observer reported being of mixed handedness. All were naïve as to the purpose of the experiments, although most had participated in other psychophysical experiments. None had participated in Experiments 1–4. All reported normal or corrected to normal vision. Each observer was paid \$7.50 for participating in each of the two blocks of the experiment. Four additional observers completed the first block of the experiment, either SJ or TOJ, but chose not to complete the second block and were thus excluded from the analysis.

### *III.7.2.2 Procedure*

The method is similar to that of Experiment 1, but one, two, four, six, eight or ten of the white ring-shaped cues could appear in a range of 6–8° of the fixation point. One of the cues occurred concentric with one of the targets, but the other target appeared at a location where no cue appeared. The centers of the cues were located at least 3° from each other and the uncued target, but only those trials in which the targets were separated by at least 7° were analyzed further. To reduce the number of conditions, all of the cues were presented 150 ms before the cued target. This CLT was chosen to be within the window of maximal exogenous attention and to be consistent with TOJ experiments in the literature, but as seen in Experiment 1, larger effects might be expected at CLTs somewhat shorter than this. The stimulus time course is illustrated in Figure 11.



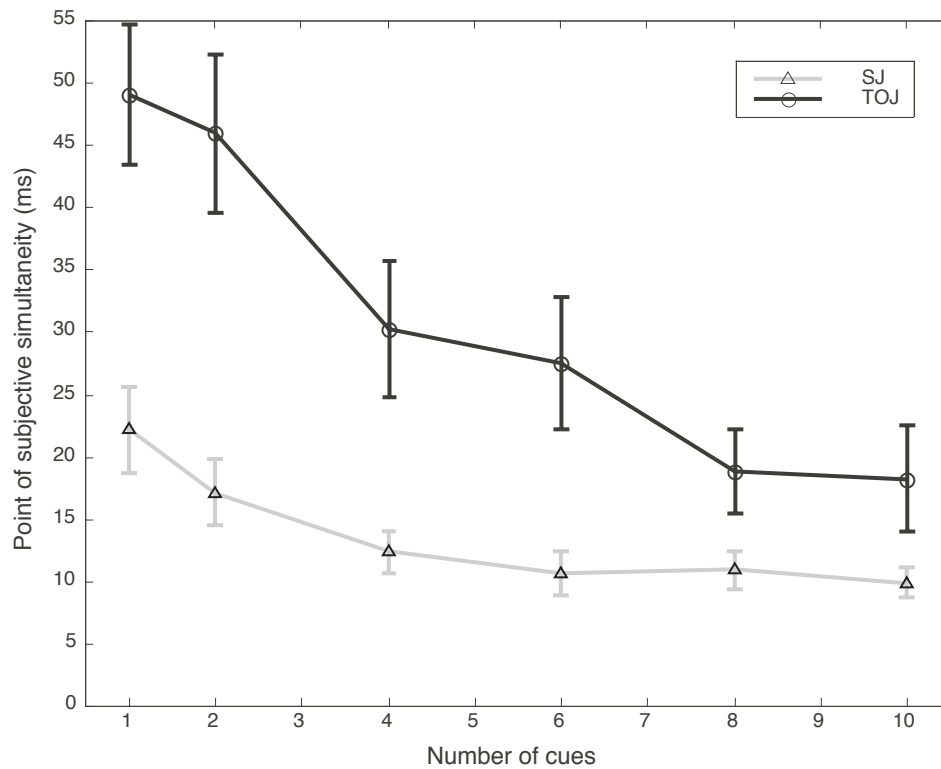
**Figure 11.** An example stimulus frame time sequence for Experiment 5. The cue lead time (CLT) between the cue and the cued target and stimulus onset asynchrony (SOA) between the cued and uncued targets are shown. The illustrated SOA is negative because the uncued target led the cued target. The figure is not to scale—for clarity, the sizes of the targets and cues have been exaggerated relative to their distances.

### III.7.3 Results

In approximately 23% of the trials, the two targets had been presented in locations less than  $7^\circ$  apart. In order to conform with the methodology of previous experiments, these trials were excluded from further analysis. Of the remaining trials, an average of 6.4% for the SJ block and 5.7% for the TOJ block for each observer

were excluded from further analysis due to delayed reaction times, following the procedure described in Experiment 1. Also, one observer's parameters at one CLT had to be eliminated from further analysis due to the failure to determine the error estimate, as described in Experiment 1.

The PSSs were calculated for the SJ and TOJ as in Experiment 1 and appear in Figure 12 and Table 5. The PSSs determined from both the SJ and TOJ decrease with an increasing number of cues. The PSSs for the TOJ were significantly larger and different than those for the SJ, and decreased to a greater extent as the number of cues increased. All of the PSSs for both the SJ and TOJ differed significantly from zero. The PSSs for both the SJ and TOJ seemed to decrease towards an asymptote at a large number of cues, though not conclusively for the TOJ.



**Figure 12.** Shifts in the points of subjective simultaneity (PSS) for the simultaneity (SJ) and order (TOJ) judgments caused by the multiple exogenous cues with a cue lead times of 150 ms in Experiment 5. The PSSs for the SJ and TOJ all differed significantly from zero at each number of cues, and from each other for all but ten cues, at which they were only marginally different. Error bars indicate the standard error of the weighted means.

**Table 5.** Mean shifts in the points of subjective simultaneity (PSS) and their significance levels produced by the multiple exogenous cues in Experiment 5.

Number of cues	1	2	4	6	8	10
PSS <sub>SJ</sub> (ms)	21.8	16.8	12.0	10.3	10.6	9.6
$p(\text{PSS}_{\text{SJ}} = 0)$	<.001	<.001	<.001	<.001	<.001	<.001
PSS <sub>TOJ</sub> (ms)	49.1	45.9	30.2	27.5	18.8	18.2
$p(\text{PSS}_{\text{TOJ}} = 0)$	<.001	<.001	<.001	<.001	<.001	<.001
$p(\text{PSS}_{\text{SJ}} = \text{PSS}_{\text{TOJ}})$	<.001	<.001	.005	.006	.04	.06

### III.7.4 Discussion

That the PSSs were not zero with a large number of cues indicates that there is a non-attentional component to shifts in the PSS. However, there is also a likely contribution of attention, as indicated by the decrease in PSS as attention was diluted with the increasing number of cues.

The condition with only one cue should be consistent with the results from Experiment 1 but instead might appear to be smaller. However, the closest CLT tested in Experiment 1 was 125 ms, compared to the 150 ms used in this experiment. The effects declined sharply at later CLTs in Experiment 1, so the results of the two experiments are not clearly discrepant.

## **III.8 General discussion of Experiments 1–5**

### III.8.1 Main findings

There are several striking results from the experiments. Foremost is the difference between the SJ and TOJ observed in all of the experiments. The attentional acceleration hypothesis predicts identical results for the two judgments, and that there is a difference suggests that some aspect of the decision process differs between them. The nature of the decision process will be investigated in Parts IV–V.

The second main finding is that the SJ effects are small or null for attentional cues that do not involve abrupt onsets at the location of the target stimulus in Experiments 2–3. Either these types of cues do not produce much attentional acceleration, or else some aspect of the SJ decision mechanism exactly cancels their effects.

The third main finding is that the attentional effects decreased but reached a finite asymptote for a large number of cues in Experiment 5. This suggests that there

are both attentional and non-attentional contributions of the exogenous cues on the SJ and TOJ.

The last finding concerns the nature of the exogenous cues. These types of cues can cause effects on the SJ and TOJ with both color- and luminance-defined abrupt onset cues, with the effects of the isoluminant cues delayed relative to the luminance-defined cues.

### III.8.2 The difference between exogenous and endogenous cues

Given that the tip of the central arrow used in Experiment 2 (and even the pupils in Experiment 3) was always at least slightly closer to the cued target than to the uncued target, one might wonder if the arrow tip acted as an exogenous cue at a distance. Several researchers (e.g. Downing & Pinker, 1985; Egly & Homa, 1991; Eriksen, Pan, & Botella, 1993; Henderson & Macquistan, 1993; Hughes & Zimba, 1987; Rizzolatti, Riggio, Dascola & Umiltà, 1987; Shulman, Sheehy & Wilson, 1986) have found that the effects of an exogenous attentional cue decreases as the distance between the cue and target increases. Without systematically varying the distance between the cue and the target, then, it is impossible to rule out that the endogenous and gaze-directed cues used in these experiments yielded different results from the exogenous cues only because they were farther from the targets. The effects of the cue-target distance was not investigated, and there may or may not be a qualitative change in the cue action when it is located beyond a critical distance. It is unknown whether the exogenous cues must be located at the target location or merely nearby in order to significantly effect temporal judgments.



### III.8.3 Eye movements

Although eye movements were not monitored in the experiments, it is unlikely that eye movements would affect the conclusions. First, many of the observers in the experiments were graduate students with extensive psychophysical performance and have been trained to maintain fixation. Second, Stelmach and Herdman (1991) monitored eye movements in one of their attentional TOJ experiments but did not notice a difference when eye movements were excluded. Third, and perhaps most importantly, the most significant attentional effects were observed in the present experiments at very short CLTs. If all results employing CLTs of greater than 150 ms were excluded, the general conclusions would still hold. Given that the stimuli occurred in randomly determined positions in the visual field a distance of  $7^\circ$  from the fixation point, it would not be possible to make a saccade to the cued location before the stimulus appeared at that location. Crawford and Muller (1992), for example, found that saccadic latencies to attentional cues averaged over 200 ms. Saccades initiated immediately upon the arrival of the cue could disrupt the perception of the stimuli and might affect response biases, but they would not be expected to produce a pattern of results consistent with the attentional acceleration hypothesis.

## IV Descriptions of the Decision Mechanisms

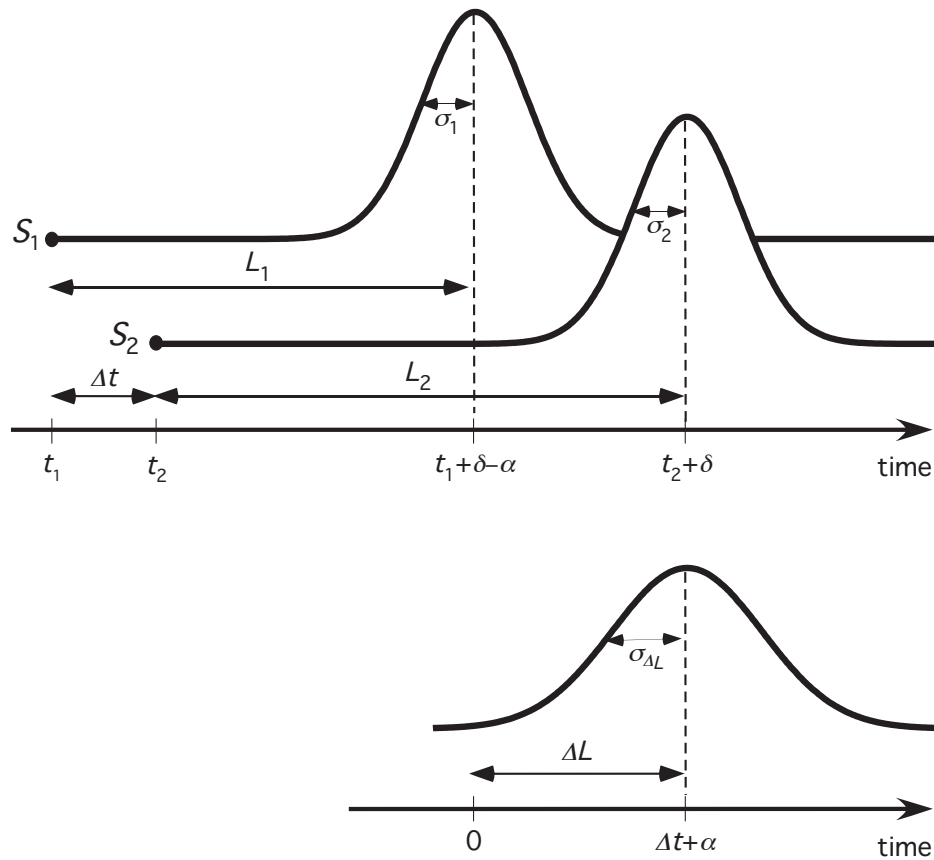
### IV.1 Model framework

The framework of the model will be the same as Sternberg and Knoll's (1973) general model, but with slightly different notation. In their model, transmission of the stimuli through the nervous system is thought to cause their arrival times at the central decision mechanism to be delayed and dispersed relative to their onset times. It is assumed that a central mechanism makes a decision on the simultaneity or relative order of two stimuli based on these arrival times. If the decision mechanism does not compensate, then the variability in the arrival latencies could be the source of error between the objective and subjective simultaneity.

The two stimuli are assumed to be independent, meaning that the nature or timing of one stimulus does not interfere with the other. Even though latencies of sensory processes should have asymmetrical probability distributions including a minimum possible latency, for simplicity, the central arrival latencies  $L_1$  and  $L_2$  are assumed to be independent and normally distributed random variables with means  $\mu_1$  and  $\mu_2$  and variances  $\sigma_1^2$  and  $\sigma_2^2$ . The mean arrival latency for each stimulus should equal the stimulus onset time plus a delay. Identical stimuli should have, on average, identical arrival delays,  $\delta$ . However, the prior entry hypothesis states that the delay for an attended stimulus should be reduced relative to that of an unattended stimulus. If we define  $S_1$  as the attended stimulus and  $S_2$  as the unattended stimulus, then the mean arrival times are  $\mu_1 \equiv t_1 + \delta - \alpha$ , where  $\alpha$  is the latency reduction due to the hypothesized acceleration effect of attention, and  $\mu_2 \equiv t_2 + \delta$ . Since the difference between two normal distributions is also normally distributed, we have  $\Delta L \equiv L_2 - L_1 \sim \mathcal{N}(t, \mu_{\Delta L}, \sigma_{\Delta L})$ , where  $\mu_{\Delta L} = \Delta t + \alpha$ ,  $\Delta t \equiv t_2 - t_1$ ,  $\sigma_{\Delta L}^2 = \sigma_1^2 + \sigma_2^2$ , and

$$\mathcal{N}(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \text{ The goal then is to infer } \alpha \text{ and } \sigma_{\Delta L} \text{ from the}$$

distribution of observer responses over a range of  $\Delta t$  values. To reject the prior entry hypothesis then is to accept the null hypothesis that  $\alpha = 0$ . Hereafter  $\sigma_L$  will be referred to simply as  $\sigma$  and will be assumed to consume all of the variability in the central latency differences, including both those arising from transmission dispersion as well as those contributed by central mechanisms.



**Figure 13.** Schematic illustrating the relationships among presentation times,  $t_1$  and  $t_2$  of two stimuli  $S_1$  and  $S_2$ , the distributions of their latencies  $L_1$  and  $L_2$ , and the distribution of the difference between their latencies  $\Delta L$ .

The problem with many previous studies of attention and prior entry was that they all used the PSS, the point at which the two possible orders of a pair of stimuli

are reported with equal frequency, as a measure of attentional latency reduction. As we will see, cognitive factors such as criterion shifts and response biases can have dramatic effects on the PSS measured in a TOJ experiment. These factors can be inferred from skew or asymmetry present in the psychometric function.

## **IV.2 Models of the decision mechanism**

Sternberg and Knoll (1973) presented a number of different models of the decision process. The attention-switching models (their Models 4 and 5) have had some success (e.g. Allan, 1975), but they make the assumption that only one stimulus can be attended at once, and that this assumption is violated within the visual domain (see e.g. Baron, 1973), so these models will not be considered. Instead I will investigate in detail the first of the models they review, the triggered-moment model. Their Model 1, the deterministic decision rule, is a special case of the triggered-moment model, as explained below. The triggered-moment model may also be used to model responses in a SJ as well as TOJ task.

The main difference between the model I will present and those presented in Sternberg and Knoll (1973) is that I have not assumed unbiased responses. For simplicity, Sternberg and Knoll presented models in which observers would respond equally likely to one stimulus or the other when uncertain about the actual order of the stimuli, but I have lifted this requirement and included the response bias as a parameter to be determined.

The triggered-moment model is not the only model from Sternberg and Knoll (1973) worth considering. Indeed, preliminary investigations of the perceptual moment model described in the Appendix, Section A.3, suggest that it may in certain circumstances provide a superior account of the data than does the triggered-moment model. However, the conclusions drawn from the two models are qualitatively very similar, and the triggered-moment theory is theoretically and computationally simpler, so only results from this model will be presented here. Ulrich (1987) showed

that the triggered-moment and perceptual moment models were inconsistent with observers' performances in a ternary response task, but this should be considered in light of the objections to the ternary response task raised above. Further, his models did not account for the influence of response biases.

#### IV.2.1 Triggered-moment model

##### *IV.2.1.1 Introduction*

The triggered-moment model assumes that observers have discrete moments of time perception, and that all stimuli that occur within one of these moments are determined to be simultaneous. The occurrence of the first stimulus triggers the beginning of such a moment, and if the other stimulus appears before a threshold time  $\tau > 0$  has elapsed, the two stimuli will be perceived as simultaneous.

##### *IV.2.1.2 Simultaneity judgment*

The simultaneity decision rule is

$$P(\text{"simultaneous"}) = \begin{cases} 1 & \text{if } |\Delta L| \leq \tau \\ 0 & \text{if } |\Delta L| > \tau. \end{cases} \quad (2)$$

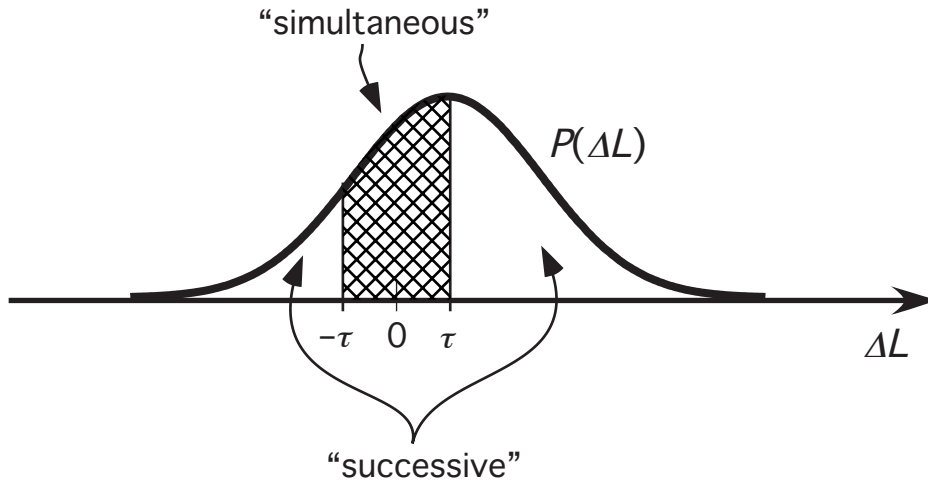
This is illustrated in Figure 14. As is always the case with a two-alternative forced choice task,  $P(\text{"successive"}) = 1 - P(\text{"simultaneous"})$ . The response probability can be calculated,

$$\begin{aligned} P(\text{"simultaneous"}) &= P\{|\Delta L| \leq \tau\} = \int_{-\tau}^{\tau} \mathcal{N}(x, \Delta t + \alpha, \sigma) dx \\ &= \Phi(\tau, \Delta t + \alpha, \sigma) - \Phi(-\tau, \Delta t + \alpha, \sigma), \end{aligned} \quad (3)$$

with the cumulative normal distribution function,

$$\Phi(x, \mu, \sigma) \equiv \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^x e^{-(t-\mu)^2/2\sigma^2} dt . \quad (4)$$

Note that it is not necessary to have symmetric simultaneity criteria. For example, the observer could respond “simultaneous” if  $-\tau_1 \leq \Delta L \leq \tau_2$  and “successive” otherwise. However, symmetric criteria  $\tau = \tau_1 = \tau_2$  can be used with no loss of generality—this case is indistinguishable from that with asymmetric criteria and an additive shift in  $\alpha$ . Note also that the maximum probability of reporting “simultaneous,” analogous to the point of subjective simultaneity in a TOJ since the simultaneity function is symmetric, is independent of the choice of criterion  $\tau$ . If it can be assumed that the criteria are symmetrically chosen as  $\pm\tau$ , then this independence implies that the SJ should provide a more direct and robust estimator of  $\alpha$ .



**Figure 14.** Illustration of the decision mechanism for the triggered-moment simultaneity judgment. The decision operates on the difference in latency  $\Delta L$  between two stimuli. The observer reports that the two stimuli were simultaneous if  $|\Delta L| < \tau$ . Otherwise, the observer reports the stimuli as successive.

### IV.2.1.3 Order judgment

For the TOJ, the observer reports that the attended stimulus occurs first if  $\Delta L > \tau$  and that the unattended stimulus occurs first if  $\Delta L < -\tau$ . If  $-\tau \leq \Delta L \leq \tau$ , the observer is unable to make an order determination and reports “ $S_1$  first” with probability  $\beta$ . If  $\beta = 0.5$ , the observer is unbiased, and this model corresponds exactly to Model 3 in Sternberg and Knoll (1973). The decision rule, illustrated in Figure 15, is

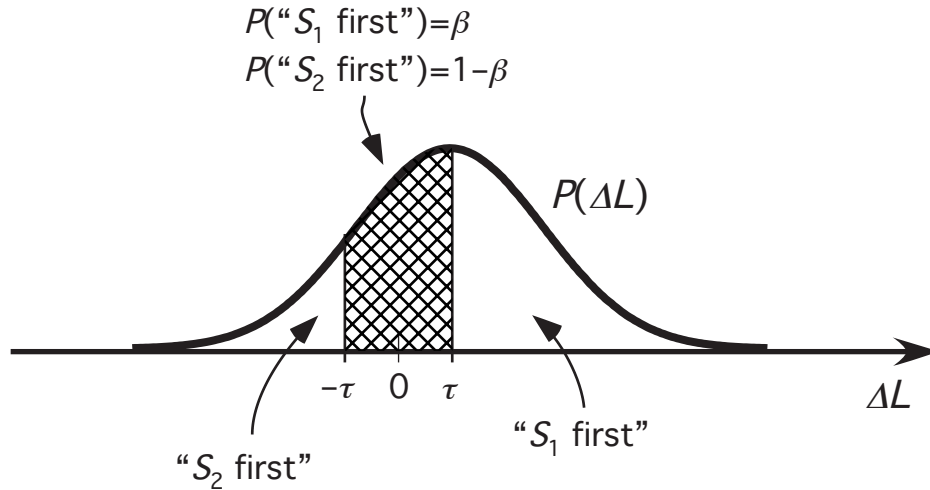
$$P(\text{“}S_1 \text{ first”}) = \begin{cases} 1 & \text{if } \Delta L > \tau \\ \beta & \text{if } |\Delta L| \leq \tau \\ 0 & \text{if } \Delta L < -\tau. \end{cases} \quad (5)$$

The response probability can therefore be calculated,

$$\begin{aligned} P(\text{“}S_1 \text{ first”}) &= P\{\Delta L > \tau\} + \beta P\{|\Delta L| \leq \tau\} \\ &= \int_{\tau}^{\infty} \mathcal{N}(x, \Delta t + \alpha, \sigma) dx + \beta \int_{-\tau}^{\tau} \mathcal{N}(x, \Delta t + \alpha, \sigma) dx \\ &= 1 + (\beta - 1)\Phi(\tau, \Delta t + \alpha, \sigma) - \beta\Phi(-\tau, \Delta t + \alpha, \sigma). \end{aligned} \quad (6)$$

Although this model assumes that a determination of successiveness is necessary for determination of order, it should be noted this model contains other models that do not have this requirement. For example, if  $\tau = 0$ , then this model reduces to the deterministic decision rule (Sternberg and Knoll’s, 1973, Model 1) in which observer reports “ $S_1$  first” if  $\Delta L > 0$ . If  $\Delta L$  is normally-distributed, then the deterministic decision rule yields the cumulative normal psychometric function that was used to superficially examine the data in Experiments 1–5 above. In addition, a decision process biased in favor of the attended stimulus, reporting for instance “ $S_1$  first” if  $\Delta L > -\tau$  and “ $S_2$  first” otherwise, is indistinguishable from the triggered-

moment model with  $\tau = 0$  and an additive shift in  $\alpha$ . Also, as in the simultaneity decision, the case with symmetric decision criteria  $\pm\tau$  is indistinguishable from that with asymmetric criteria and a shift in  $\alpha$ .



**Figure 15.** Illustration of the decision mechanism for the triggered-moment temporal order judgment. The decision operates on the difference of the latencies  $\Delta L$  of two stimuli,  $S_1$  and  $S_2$ . If  $\Delta L > \tau$ , then the observer will respond that  $S_1$  occurred first. If  $\Delta L < -\tau$ , then the observer will respond that  $S_2$  occurs first. If  $|\Delta L| < \tau$ , then the observer is unable to determine the order of the stimuli and responds with probability  $\beta$  that  $S_1$  occurred first. If  $\beta = 0.5$ , the observer is unbiased.



## V Model Testing

### V.1 Introduction

The main goal of this dissertation is to test whether or not the attentional acceleration hypothesis is consistent with the data, that is, whether models with  $\alpha = 0$  are more consistent than models where  $\alpha$  is allowed to vary. To put it another way, is it necessary to add another parameter  $\alpha$  in order to explain the data? A second goal is to determine whether the SJ and TOJ decisions operate on the same  $\Delta L$  distribution, or upon distinct distributions. In other words, does  $\alpha_{\text{SJ}} = \alpha_{\text{TOJ}}$  and  $\sigma_{\text{SJ}} = \sigma_{\text{TOJ}}$ ?

Following MacKay (1992), there are two steps to testing hypotheses. First, assume that one hypothesis  $\mathcal{H}_k$  is true and find the most probable parameters for that model. The second step is to infer which of the various models  $\mathcal{H}_k$  is most plausible given the data.

### V.2 Model fitting

Let  $\mathbf{w}$  be vector of parameters for  $\mathcal{H}_k$ , e.g.  $\mathbf{w} = (\alpha, \sigma, \tau, \beta)$  for the triggered-moment or perceptual-moment TOJ hypothesis. We want to find the parameters  $\hat{\mathbf{w}}$  that are most probable given the data  $\mathcal{D}$  (defined below) and  $\mathcal{H}_k$ . That is, we want to maximize the posterior probability of the parameters  $\mathbf{w}$ , given by Bayes' rule as

$$P(\mathbf{w}|\mathcal{D}, \mathcal{H}_k) = \frac{P(\mathcal{D}|\mathbf{w}, \mathcal{H}_k)P(\mathbf{w}|\mathcal{H}_k)}{P(\mathcal{D}|\mathcal{H}_k)}. \quad (7)$$

Given that the evidence  $P(\mathcal{D}|\mathcal{H}_k)$  is independent of  $\mathbf{w}$ , and assuming that  $P(\mathbf{w}|\mathcal{H}_k)$ , the prior probability of the parameters, is uniform, maximizing the

posterior probability is equivalent to maximizing the likelihood function,

$$\mathcal{L} \equiv P(\mathcal{D} | \mathbf{w}, \mathcal{H}_k), \text{ i.e. finding } \hat{\mathbf{w}} \text{ that satisfies } \mathcal{L}_{max} \equiv P(\mathcal{D} | \hat{\mathbf{w}}, \mathcal{H}_k).$$

In both the SJ and TOJ, the observer has two response alternatives  $R_0$  and  $R_1$ , where  $R_0$  is “successive” for the SJ and “ $S_2$  first” for the TOJ, and  $R_1$  is “simultaneous” for the SJ and “ $S_1$  first” for the TOJ. The psychometric function is sampled by  $n_i$  independent repetition trials for each of the  $m$  conditions having different values of  $\Delta t$ . If  $X_{ij}$  represents the response on the  $j$ th repetition of condition  $i$ , and

$$X_{ij} = \begin{cases} 1 & \text{if } R_1 \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

then  $X_{ij}$  is a Bernoulli random variable, with

$$P\{X_{ij} = x\} = p_i^x (1 - p_i)^{1-x}, \quad (9)$$

where  $p_i \equiv P\{X_{ij} = 1 | \Delta t_i, \mathbf{w}\}$  at  $\Delta t_i$  the SOA for condition  $i$ . The data set for each

observer,  $\mathcal{D} \equiv \left\{ Y_i \equiv \sum_{j=1}^{n_i} X_{ij} \right\}_{i=1}^m$ , is composed of the values  $Y_i$ , one for each of the  $m$

conditions, that are the total number of  $R_1$  responses among the  $n_i$  repetitions of each condition. The likelihood function is given by

$$\mathcal{L} = \prod_{i=1}^m \prod_{j=1}^{n_i} p_i^{X_{ij}} (1 - p_i)^{1-X_{ij}}. \quad (10)$$

Maximizing  $\mathcal{L}$  is equivalent to minimizing  $-\log \mathcal{L}$ , with

$$\log \mathcal{L} = \sum_{i=1}^m Y_i \log p_i + (n_i - Y_i) \log(1 - p_i). \quad (11)$$

Assuming the likelihood function is can be approximated by a normal distribution near its maximum, the variances of the parameters  $\mathbf{w}$  can by obtained from the covariance matrix  $\mathbf{A}^{-1}$ , where  $\mathbf{A} \equiv -\nabla^2 \log \mathcal{L}$  is the Hessian. The calculation of  $\mathbf{A}$  for the triggered-moment and perceptual-moment theories is detailed in the Appendix, Section A.3.

### V.3 Model comparison

The goal of the second step of inference is to infer which model  $\mathcal{H}_k$  is most plausible given the data. There are two procedures for this that are useful for different purposes.

#### V.3.1 Nested hypothesis testing with the likelihood ratio

If the two hypotheses are nested such that the simpler model can be obtained from the more complex model by eliminating one or more of the parameters, then the likelihood functions of the two models can be compared directly using a likelihood ratio test (Rao, 1973). The primary goal of this dissertation is to test whether the data forces the rejection the null hypothesis  $\mathcal{H}_0: \alpha = 0$  versus the alternative hypothesis

$\mathcal{H}_1: \alpha \neq 0$  for the various models. Let  $\hat{\mathbf{w}}_1 \equiv (\hat{\alpha}_1, \hat{\sigma}_1, \hat{\tau}_1, \hat{\beta}_1)$  and  $\hat{\mathbf{w}}_0 \equiv (\alpha = 0, \hat{\sigma}_0, \hat{\tau}_0, \hat{\beta}_0)$

maximize the likelihood functions, i.e.  $\mathcal{L}_{1,\max} \equiv P(\mathcal{D}|\hat{\mathbf{w}}_1, \mathcal{H}_1)$  and

$\mathcal{L}_{0,\max} \equiv P(\mathcal{D}|\hat{\mathbf{w}}_0, \mathcal{H}_0)$  respectively, then

$$\lambda \equiv \frac{\mathcal{L}_{0,\max}}{\mathcal{L}_{1,\max}} \sim \chi_1^2, \quad (12)$$

where  $\chi_1^2$  is the chi-square distribution with one degree of freedom. We can reject  $\mathcal{H}_0$  if  $-2 \log \lambda > \chi_1^2$  at a chosen two-sided probability level. Given that data from different observers is independent,  $\mathcal{H}_0$  can be tested incorporating data from all observers using the product of the individual likelihood ratios. Calculating  $\lambda_k$  as above for each of  $N$  observers, we can reject  $\mathcal{H}_0$  if

$$-2 \sum_{i=1}^N \log \lambda_k > \chi_N^2. \quad (13)$$

### V.3.2 Bayesian inference

An alternative to the nested hypothesis test for model comparison is Bayesian inference. This can be used to compare both nested and non-nested models. The posterior probability of each hypothesis is

$$P(\mathcal{H}_k | \mathcal{D}) \propto P(\mathcal{D} | \mathcal{H}_k) P(\mathcal{H}_k), \quad (14)$$

and assuming that there is no a priori preference for any particular model,  $P(\mathcal{H}_k)$  should be uniform for each  $\mathcal{H}_k$ , and we can rank the models by evaluating  $P(\mathcal{D} | \mathcal{H}_k)$ , the probability of the observations given each  $\mathcal{H}_k$ . The evidence is given by

$$P(\mathcal{D} | \mathcal{H}_k) = \int P(\mathcal{D} | \mathbf{w}, \mathcal{H}_k) P(\mathbf{w} | \mathcal{H}_k) d\mathbf{w}. \quad (15)$$

If  $\mathbf{w}$  is  $d$ -dimensional and if the posterior  $\mathcal{L} = P(\mathbf{w} | \mathcal{D}, \mathcal{H}_k)$  is assumed to have a strong peak near  $\hat{\mathbf{w}}$ , then the evidence can be approximated as (Gull, 1988; MacKay, 1992):

$$P(\mathcal{D}|\mathcal{H}_k) \equiv \mathcal{L}_{\max} P(\hat{\mathbf{w}}|\mathcal{H}_k) (2\pi)^{d/2} \det(\mathbf{A})^{-1/2}. \quad (16)$$

Using this, it is easy to compare the relative probabilities of two models. The probability of the model given the data from all observers, given that the observers are independent, is simply the product of the probabilities from each observer. Illustrating this is a comparison between the triggered-moment ( $\mathcal{H}_{\text{TM}}$ ) and perceptual-moment ( $\mathcal{H}_{\text{PM}}$ ) hypotheses in the Appendix, Section A.3.  $\mathcal{H}_{\text{TM}}$  and  $\mathcal{H}_{\text{PM}}$  have the same parameters that can reasonably be assumed to have identical priors. This comparison is not associated with a statistical test, and only the relative probabilities of two models can be determined. For  $N$  independent observers, the triggered-moment theory should be favored if

$$\prod_{i=1}^N \frac{\mathcal{L}_{\max, \text{TM}, i} \det^{-1/2} \mathbf{A}_{\text{TM}, i}}{\mathcal{L}_{\max, \text{PM}, i} \det^{-1/2} \mathbf{A}_{\text{PM}, i}} \gg 1. \quad (17)$$

### V.3.3 Nested hypothesis testing with Bayesian inference

Like the likelihood ratio test, Bayesian inference can also be used to test nested hypotheses. As an example, one nested hypothesis to test is whether or not the SJ and TOJ decisions are based on the same latency distribution  $\Delta L$ . To test this, the data is fit to two models. The first model  $\mathcal{H}_{=}$  assumes that  $\Delta L_{\text{SJ}} = \Delta L_{\text{TOJ}}$ , i.e. it has five parameters:  $\alpha = \alpha_{\text{SJ}} = \alpha_{\text{TOJ}}$ ,  $\sigma = \sigma_{\text{SJ}} = \sigma_{\text{TOJ}}$ ,  $\tau_{\text{SJ}}$ ,  $\tau_{\text{TOJ}}$  and  $\beta$ . The second model  $\mathcal{H}_{\neq}$  assumes that  $\Delta L_{\text{SJ}} \neq \Delta L_{\text{TOJ}}$  and has seven parameters,  $\alpha_{\text{SJ}}$ ,  $\alpha_{\text{TOJ}}$ ,  $\sigma_{\text{SJ}}$ ,  $\sigma_{\text{TOJ}}$ ,  $\tau_{\text{SJ}}$ ,  $\tau_{\text{TOJ}}$  and  $\beta$ . We can assume that the priors on the parameters are equal between the models, and, with no intuition, set the probability distribution of the parameters to be uniform within a specified range, e.g.  $\alpha \in [-\alpha_{\max}, \alpha_{\max}]$ ,  $\sigma \in [0, \sigma_{\max}]$ ,  $\tau \in [0, \tau_{\max}]$ , and

$\beta \in [0,1]$ . Given that the parameters are uniform and independent, their joint prior distributions are given by  $P(\hat{\mathbf{w}}|\mathcal{H}_=) = (2\alpha_{\max}\sigma_{\max}\tau_{\max}^2)^{-1}$  and  $P(\hat{\mathbf{w}}|\mathcal{H}_{\neq}) = (2\alpha_{\max}\sigma_{\max}\tau_{\max}^2)^{-2}$ . These priors can then be used in Equation 16. If the product of the ratios  $P(\mathcal{D}|\mathcal{H}_=)/P(\mathcal{D}|\mathcal{H}_{\neq})$  for the  $N$  observers is very small, then  $\mathcal{H}_=$  can be rejected.

#### V.3.4 Comparisons between the two types of hypothesis tests

Both the likelihood ratio test and the Bayesian inference test can be used to test nested hypotheses, but these two tests can give different results. Increasing the number of parameters in a model increases the accuracy in describing the data but also increases the complexity of the model. Each of the two types of hypothesis tests tries to determine, in different ways, whether the introduction of an additional parameter is justified. The likelihood ratio test does so by accounting for the increased number of degrees of freedom in the more complex model. The Bayesian inference test compares the reduction of parameter space between the two models. Given that the a priori expected range of one of the parameters in the model can be quite large, and that a good fit to the data places narrow limits on the model parameters, the penalty for additional parameters in the model can be quite severe. The model is fit to data from each observer independently, so the penalty for additional parameters in the model is multiplied with additional observers. Inferences employing the Bayesian inference method must be qualified by the choice of subjective priors for the parameters, but the sensitivity of the result based on the assumptions can be quantified. The likelihood ratio test behaves differently as the number of observers increases. Because additional degrees of freedom become less significant in the chi-square test as the total number of degrees of freedom increases, and the contribution of each additional data point to the likelihood ratio remains

constant, the likelihood ratio test increasingly favors the more complex model as the amount of data increases. For this reason, both the likelihood ratio and the Bayesian inference tests will be performed below to test the main hypothesis involving only a one degree of freedom per observer difference between the null and alternative hypotheses, but only Bayesian inference will be used for subsequent tests.

## V.4 Testing the attentional acceleration hypothesis

### V.4.1 Introduction

The primary goal of this dissertation is to test the null hypothesis  $\mathcal{H}_0$  that  $\alpha = 0$ , i.e. that there is no significant attentional acceleration effect. Following the procedure described above, I will test this null hypothesis for each experiment, judgment and condition using both the likelihood ratio test and Bayesian inference.

### V.4.2 Results

#### V.4.2.1 *Experiment 1. Exogenous cues.*

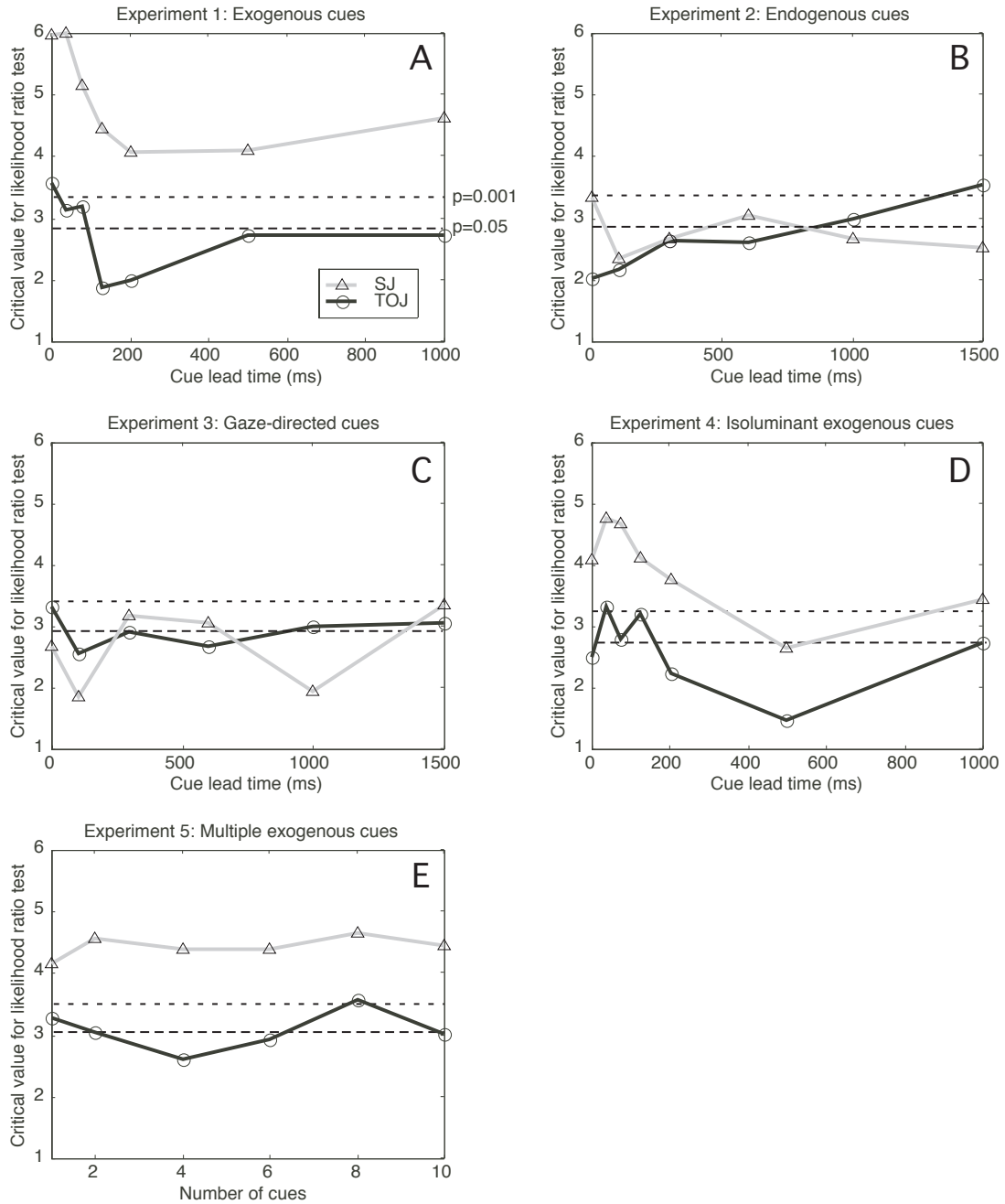
The results of the two tests on the data from Experiment 1 are shown in Figures 16A and 17A, and listed in Table 6. For both the likelihood ratio test and the Bayesian inference test,  $\mathcal{H}_0$  can be confidently rejected for the SJ at all CLTs. The TOJ gives mixed results for the two tests. The shortest three CLTs significantly support the rejection of  $\mathcal{H}_0$ , while  $\mathcal{H}_0$  is preferred at all CLTs for the Bayesian inference test.

For this and subsequent experiments, the choice of the prior distribution for the parameters did not significantly affect the results of the Bayesian inference test. When testing  $\mathcal{H}_0$  versus  $\mathcal{H}_1$ , only the prior on  $\alpha$  matters—the other priors are identical in the two models and cancel out in the comparison. In the following comparisons,  $\alpha_{\max}$ , as defined in Section V.3.3, was set to 200 ms. This is a liberal

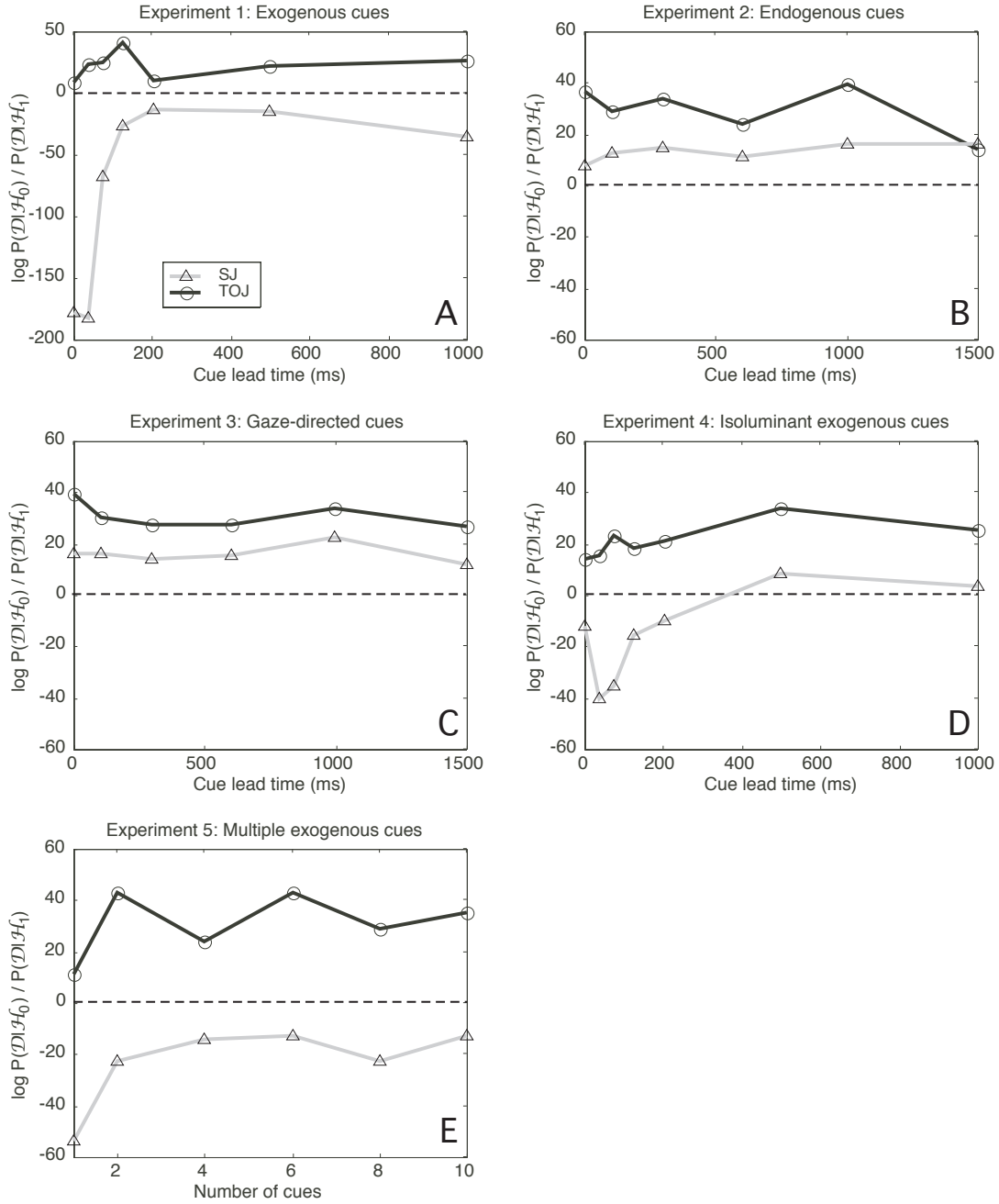
prior distribution, but even making it unreasonably strict (less than 50 ms) did not change the results significantly. Any differences between reasonable choices for the prior distributions were overwhelmed by the magnitude of the likelihood ratios.

It should be noted that for many of the observers, in this and subsequent experiments, the determinant of the Hessian was, for reasons to be investigated, non-positive at the best fit to the TOJ data. This is a violation of the assumption behind Equation 16 that the likelihood function is Gaussian near its maximum. Therefore, when one of the model fits for either  $\mathcal{H}_0$  or  $\mathcal{H}_1$  produced a non-positive determinant of the Hessian, then this point was dropped for both from the comparison in the Bayesian inference test. The number of observers for each CLT whose data was omitted in this manner is listed in Table 6. An alternative solution would be to numerically integrate Equation 15 rather than using the approximation of Equation 16, but in most cases there was enough data remaining to permit a reasonable comparison.





**Figure 16.** Results of likelihood ratio test for Experiments 1 (A) through Experiment 5 (E). The critical values for the  $\chi^2$  test from Equation 13 are plotted. The lower of the two dashed lines for each plot indicates the  $p = .05$  significance level. If the critical value is not greater than this dashed line, then the null hypothesis  $\mathcal{H}_0: \alpha = 0$  cannot be rejected. The upper dashed line indicates the  $p = .001$  significance level.



**Figure 17.** Bayesian inference model comparison test for Experiments 1 (A) through Experiment 5 (E). The natural logarithm of the product across observers of the ratio  $P(\mathcal{D}|\mathcal{H}_0)/P(\mathcal{D}|\mathcal{H}_1)$  is plotted. If the product of ratios is larger than one, indicated by the dashed line, the null hypothesis  $\mathcal{H}_0: \alpha = 0$  is preferred over the alternative  $\mathcal{H}_1: \alpha \neq 0$ .

Cue lead time (ms)	0	40	75	125	200	500	1000
$p(\alpha = 0)$ for SJ	<.001	<.001	<.001	<.001	<.001	<.001	<.001
$p(\alpha = 0)$ for TOJ	<.001	.006	.004	n.s.	n.s.	.08	.09
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for SJ	-178	-182	-68.0	-25.7 (1)	-13.6	-15.0	-34.8
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for TOJ	9.01 (7)	24.0 (3)	24.8 (3)	41.0 (1)	10.9 (4)	22.4 (4)	26.8 (1)

**Table 6.** Results of the attentional acceleration tests for Experiment 1 with exogenous cues. The first two rows of the table list, for the SJ and TOJ respectively, the  $p$ -values from the likelihood ratio test (Equation 13) for accepting the null hypothesis  $\mathcal{H}_0: \alpha = 0$ . The last two rows list the likelihood ratio derived from Bayesian inference (Equation 16). A negative value indicates that the alternative hypothesis  $\mathcal{H}_1: \alpha \neq 0$  should be preferred, and a positive value supports  $\mathcal{H}_0$ . In the last two rows, the numbers in parentheses indicate the number of points that were excluded for that comparison due to a non-positive determinant of the Hessian matrix for the fit to the model for  $\mathcal{H}_0$  or  $\mathcal{H}_1$ , as described in the text

#### V.4.2.2 Experiment 2. Endogenous cues.

The results of the two tests on the data from Experiment 2 are shown in Figures 16B and 17B, and listed in Table 7. At various CLTs for the SJ and TOJ, the likelihood ratio test significantly supports the rejection of  $\mathcal{H}_0$ . However, the Bayesian inference test distinctly prefers  $\mathcal{H}_0$  for all CLTs for both the SJ and TOJ.

Cue lead time (ms)	0	100	300	600	1000	1500
$p(\alpha = 0)$ for SJ	<.001	n.s.	.1	.01	.1	n.s.
$p(\alpha = 0)$ for TOJ	n.s.	n.s.	.1	.1	.02	<.001
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for SJ	7.71	13.0	14.8	11.6	16.5	16.2
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for TOJ	36.9 (2)	29.1 (3)	33.6 (2)	23.7 (5)	39.5 (1)	13.9 (6)

**Table 7.** Results of attentional acceleration tests for Experiment 2 with endogenous cues. The first two rows of the table list, for the SJ and TOJ respectively, the  $p$ -values from the likelihood ratio test (Equation 13) for accepting the null hypothesis  $\mathcal{H}_0: \alpha = 0$ . The last two rows list the likelihood ratio derived from Bayesian inference (Equation 16). A negative value indicates that the alternative hypothesis  $\mathcal{H}_1: \alpha \neq 0$  should be preferred, and a positive value supports  $\mathcal{H}_0$ . In the last row, the numbers in parentheses indicate the number of points that were excluded for that comparison due to a non-positive determinant of the Hessian matrix for the fit to the model for  $\mathcal{H}_0$  or  $\mathcal{H}_1$ , as described in the text.

#### V.4.2.3 Experiment 3. Gaze-directed cues.

The results of the two tests on the data from Experiment 3 are shown in Figures 16C and 17C, and listed in Table 8. The results are quite similar to those from Experiment 2. At various CLTs for the SJ and TOJ, the likelihood ratio test significantly supports the rejection of  $\mathcal{H}_0$ . However, the Bayesian inference test distinctly prefers  $\mathcal{H}_0$  for all CLTs for both the SJ and TOJ.

Cue lead time (ms)	0	100	300	600	1000	1500
$p(\alpha = 0)$ for SJ	n.s.	n.s.	.007	.02	n.s.	.001
$p(\alpha = 0)$ for TOJ	.001	n.s.	.05	n.s.	.03	.02
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for SJ	16.4	16.1	14.0	15.2	22.9	12.0
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for TOJ	39.5 (2)	30.4 (3)	27.3 (4)	27.3 (3)	33.6 (4)	26.5 (5)

**Table 8.** Results of the attentional acceleration tests for Experiment 3 with gaze-directed cues. The first two rows of the table list, for the SJ and TOJ respectively, the  $p$ -values from the likelihood ratio test (Equation 13) for accepting the null hypothesis  $\mathcal{H}_0: \alpha = 0$ . The last two rows list the likelihood ratio derived from Bayesian inference (Equation 16). A negative value indicates that the alternative hypothesis  $\mathcal{H}_1: \alpha \neq 0$  should be preferred, and a positive value supports  $\mathcal{H}_0$ . In the last row, the numbers in parentheses indicate the number of points that were excluded for that comparison due to a non-positive determinant of the Hessian matrix for the fit to the model for  $\mathcal{H}_0$  or  $\mathcal{H}_1$ , as described in the text.

#### V.4.2.4 Experiment 4. Isoluminant exogenous cues.

The results of the two tests on the data from Experiment 4 are shown in Figures 16D and 17D, and listed in Table 9. The likelihood ratio test reveals that the SJ significantly supports the rejection of  $\mathcal{H}_0$  at all but the 500 ms CLT, while the TOJ only significantly supports the rejection of  $\mathcal{H}_0$  at the 40 and 75 ms CLTs. For the SJ, the Bayesian inference test distinctly prefers  $\mathcal{H}_0$  for all but the 500 and 1000 ms CLTs, while for the TOJ, the test prefers  $\mathcal{H}_1$  at all CLTs.

Cue lead time (ms)	0	40	75	125	200	500	1000
$p(\alpha = 0)$ for SJ	<.001	<.001	<.001	<.001	<.001	.08	<.001
$p(\alpha = 0)$ for TOJ	n.s.	<.001	.03	.002	n.s.	n.s.	.05
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for SJ	-11.8	-40.6	-35.1	-15.8	-10.0	8.22	3.29
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for TOJ	14.0 (4)	15.9 (4)	23.6 (3)	18.3 (3)	21.1 (4)	34.1 (1)	25.2 (2)

**Table 9.** Results of the attentional acceleration tests for Experiment 4 with isoluminant exogenous cues. The first two rows of the table list, for the SJ and TOJ respectively, the  $p$ -values from the likelihood ratio test (Equation 13) for accepting the null hypothesis  $\mathcal{H}_0: \alpha = 0$ . The last two rows list the likelihood ratio derived from Bayesian inference (Equation 16). A negative value indicates that the alternative hypothesis  $\mathcal{H}_1: \alpha \neq 0$  should be preferred, and a positive value supports  $\mathcal{H}_0$ . In the last row, the numbers in parentheses indicate the number of points that were excluded for that comparison due to a non-positive determinant of the Hessian matrix for the fit to the model for  $\mathcal{H}_0$  or  $\mathcal{H}_1$ , as described in the text.

#### V.4.2.5 Experiment 5. Multiple exogenous cues.

The results of the two tests on the data from Experiment 5 are shown in Figures 16E and 17E, and listed in Table 10. For the SJ, both the likelihood ratio test and the Bayesian inference test prefer  $\mathcal{H}_1$  at each number of cues tested. For the TOJ, the two tests differ. For the likelihood ratio test, the TOJ data significantly support the rejection of  $\mathcal{H}_0$  only for one, two or eight cues, while the Bayesian inference test prefers  $\mathcal{H}_0$  at each number of cues tested.

Number of cues	1	2	4	6	8	10
$p(\alpha = 0)$ for SJ	<.001	<.001	<.001	<.001	<.001	<.001
$p(\alpha = 0)$ for TOJ	.009	.05	n.s.	.09	<.001	.06
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for SJ	-53.4	-22.4	-14.4	-12.9	-22.5	-12.4
$\log \frac{P(\mathcal{D} \mathcal{H}_0)}{P(\mathcal{D} \mathcal{H}_1)}$ for TOJ	11.1 (6)	43.1 (3)	24.0 (5)	43.2 (2)	29.0 (6)	35.5 (4)

**Table 10.** Results of the attentional acceleration tests for Experiment 5 with multiple exogenous cues. The first two rows of the table list, for the SJ and TOJ respectively, the  $p$ -values from the likelihood ratio test (Equation 13) for accepting the null hypothesis  $\mathcal{H}_0: \alpha = 0$ . The last two rows list the likelihood ratio derived from Bayesian inference (Equation 16). A negative value indicates that the alternative hypothesis  $\mathcal{H}_1: \alpha \neq 0$  should be preferred, and a positive value supports  $\mathcal{H}_0$ . In the last row, the numbers in parentheses indicate the number of points that were excluded for that comparison due to a non-positive determinant of the Hessian matrix for the fit to the model for  $\mathcal{H}_0$  or  $\mathcal{H}_1$ , as described in the text.

#### V.4.3 Discussion of attentional acceleration hypothesis tests.

In general, the SJ data support  $\mathcal{H}_1$  when the cue was an abrupt onset at the target location (Experiments 1, 4 and 5), and supported  $\mathcal{H}_0$  otherwise (Experiments 2 and 3). An exception to this involved the isoluminant exogenous cues of Experiment 4, which supported  $\mathcal{H}_0$  for CLTs of 500 and 1000 ms. Thus, exogenous cues seem to produce attentional acceleration, surprisingly even when simultaneous with the target (at a CLT of 0 ms). Given that attentional effects are thought to take some time to develop, this supports the notion that the effects of the exogenous cues may be due to sensory facilitation rather than attention.

The contribution of sensory facilitation is further supported by the results from Experiment 5 employing multiple exogenous cues. The SJ data supports  $\mathcal{H}_1$  even when ten exogenous cues are presented at once. With this many cues, any attentional effects should be extremely diluted, while the sensory facilitation effects would still be present. On the whole, the results from the SJ tests indicate that sensory facilitation is a significant component of the apparent latency reduction. However, further effects of exogenous attention cannot be ruled out.

A non-zero attentional acceleration parameter is not necessary to explain the TOJ results. The models with  $\alpha = 0$  are still able to sufficiently account for the data for all experiments. The discrepancy between the SJ and TOJ results may be due to the additional parameter (four versus three) that the TOJ model contains. Eliminating one of them may still allow enough flexibility to fit the data well—changes in the response bias  $\beta$  may be an adequate explanation for the TOJ results. Attentional acceleration effects may well be present in the TOJ data but are not uniquely able to explain the results.

## V.5 Testing for a single latency distribution

Given that the decision models of the SJ and TOJ are similar, one might wonder whether the two decisions could be modeled as operating on a single latency distribution  $\Delta L$ , or perhaps as even sharing the same decision criterion  $\tau$ . This question will be tested with Bayesian inference (Equation 16) as outlined in Section V.3.3.

Three models will be tested against each other. The first model,  $\mathcal{H}_7$ , assumes that the SJ and TOJ operate on independent latency distributions and includes seven independent parameters:  $\alpha_{\text{SJ}}$ ,  $\alpha_{\text{TOJ}}$ ,  $\sigma_{\text{SJ}}$ ,  $\sigma_{\text{TOJ}}$ ,  $\tau_{\text{SJ}}$ ,  $\tau_{\text{TOJ}}$  and  $\beta$ . The second model,  $\mathcal{H}_5$ , assumes that the SJ and TOJ operate on the same latency distribution but involve different decision criteria. This model has five independent parameters:  $\alpha = \alpha_{\text{SJ}} =$



$\alpha_{\text{TOJ}}$ ,  $\sigma = \sigma_{\text{SJ}} = \sigma_{\text{TOJ}}$ ,  $\tau_{\text{SJ}}$ ,  $\tau_{\text{TOJ}}$  and  $\beta$ . The third model,  $\mathcal{H}_4$ , assumes that the SJ and TOJ arise from the same latency distribution and decision criterion, with the additional response bias parameter for the TOJ. This model has four independent parameters:  $\alpha = \alpha_{\text{SJ}} = \alpha_{\text{TOJ}}$ ,  $\sigma = \sigma_{\text{SJ}} = \sigma_{\text{TOJ}}$ ,  $\tau = \tau_{\text{SJ}} = \tau_{\text{TOJ}}$  and  $\beta$ .

Since the number of parameters between each of these models varies greatly, the choice of the prior distribution for the parameters becomes crucial. I compare a conservative choice of priors, with  $\alpha$ ,  $\sigma$  and  $\tau$  confined to a uniform range of 50 ms each, with a liberal choice, with each confined to a uniform range of 200 ms. The conservative choice is the smallest reasonable choice of priors that can be used, based on the data, and often the parameters even fall outside this range. Using this conservative choice of priors, we know that we are favoring the model with the larger number of parameters as much as possible, and the resulting Bayesian inference test will provide a lower bound to the ratio of the likelihoods of each model. The liberal choice of priors is the largest range one could reasonably expect, and provides the upper bound.

Table 11 lists  $P(\mathcal{D}|\mathcal{H}_i)/P(\mathcal{D}|\mathcal{H}_j)$  jointly for the SJ and TOJ over all conditions (CLTs for Experiments 1–4 or number of cues for Experiment 5) for each pair of models. Combining the conditions was justified because an examination of the values for the individual conditions suggests that, in general, the same model is favored for each condition. The more positive the number, the more the first model of the pair is preferred. The four-parameter model  $\mathcal{H}_4$  is preferred over each of the others in all experiments, independent of the choice of priors. In contrast,  $\mathcal{H}_5$  is only preferred over  $\mathcal{H}_7$  for a liberal choice of the prior distribution for the parameters. In summary, for each experimental condition, four parameters are sufficient to explain both the SJ and TOJ results.

Experiment number		1	2	3	4	5
$\log \frac{P(\mathcal{D} \mathcal{H}_4)}{P(\mathcal{D} \mathcal{H}_7)}$	min	12.4	142	127	101	72.2
	max	141	296	290	242	239
$\log \frac{P(\mathcal{D} \mathcal{H}_5)}{P(\mathcal{D} \mathcal{H}_7)}$	min	-51.2	-2.84	-13.6	-24.7	-30.0
	max	51.4	88.7	100	72	83.7
$\log \frac{P(\mathcal{D} \mathcal{H}_4)}{P(\mathcal{D} \mathcal{H}_5)}$	min	127	158	162	183	110
	max	191	215	223	253	177

**Table 11.** Joint Bayesian inference tests across all conditions for each experiment for models with different numbers of independent parameters.  $\mathcal{H}_4$  has four independent parameters:  $\alpha = \alpha_{\text{SJ}} = \alpha_{\text{TOJ}}$ ,  $\sigma = \sigma_{\text{SJ}} = \sigma_{\text{TOJ}}$ ,  $\tau = \tau_{\text{SJ}} = \tau_{\text{TOJ}}$  and  $\beta$ ;  $\mathcal{H}_5$  has five independent parameters:  $\alpha = \alpha_{\text{SJ}} = \alpha_{\text{TOJ}}$ ,  $\sigma = \sigma_{\text{SJ}} = \sigma_{\text{TOJ}}$ ,  $\tau_{\text{SJ}}$ ,  $\tau_{\text{TOJ}}$  and  $\beta$ ; and  $\mathcal{H}_7$  has seven independent parameters:  $\alpha_{\text{SJ}}$ ,  $\alpha_{\text{TOJ}}$ ,  $\sigma_{\text{SJ}}$ ,  $\sigma_{\text{TOJ}}$ ,  $\tau_{\text{SJ}}$ ,  $\tau_{\text{TOJ}}$  and  $\beta$ . The minimum and maximum values of  $P(\mathcal{D}|\mathcal{H}_i)/P(\mathcal{D}|\mathcal{H}_j)$  are shown for each pair of models, respectively depending on a conservative or liberal choice of priors for the parameters. The more positive the value, the more  $\mathcal{H}_i$  is preferred to  $\mathcal{H}_j$ .

## V.6 Examination of model fit parameters

We will now examine effects of the experimental conditions upon the most likely model parameters. The weighted mean (as defined in Equation 1) across observers of the most likely model parameters  $\mathcal{H}_k(\hat{\alpha}, \hat{\sigma}, \hat{\tau}, \hat{\beta})$  for the exogenous (Experiment 1) and isoluminant exogenous (Experiment 4) cues are shown in Figure 18. Since the model tests in Section V.5 indicate that a four-parameter model is sufficient to describe both the SJ and TOJ data, the parameters of that model are shown.

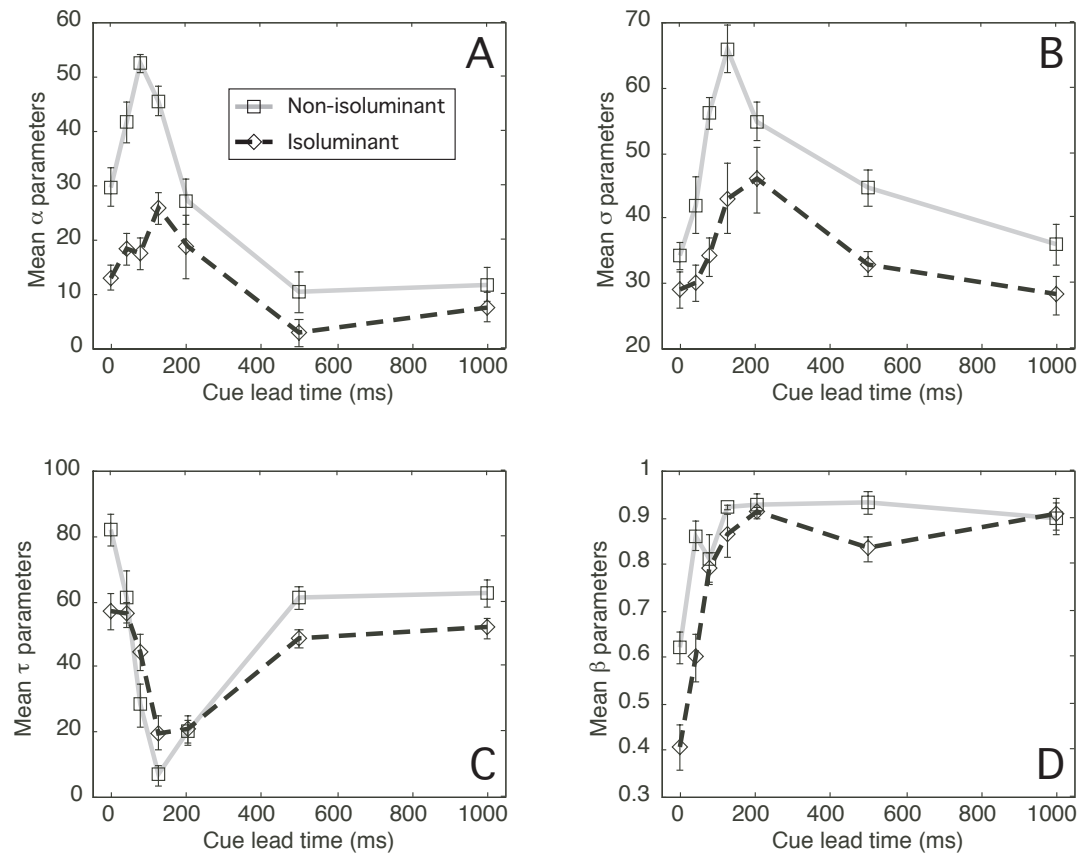
As shown in Figure 18A,  $\hat{\alpha}$  for reaches its peak quickly, by a CLT of 40 ms, for the exogenous cue, and at a somewhat later CLT, 125 ms, for the isoluminant

exogenous cue. For both cue types,  $\hat{\alpha}$  is significantly non-zero at a CLT of 0 ms, and declines to a baseline by a CLT of 500 ms. These results are qualitatively similar to the PSSs in Figures 4 and 10.

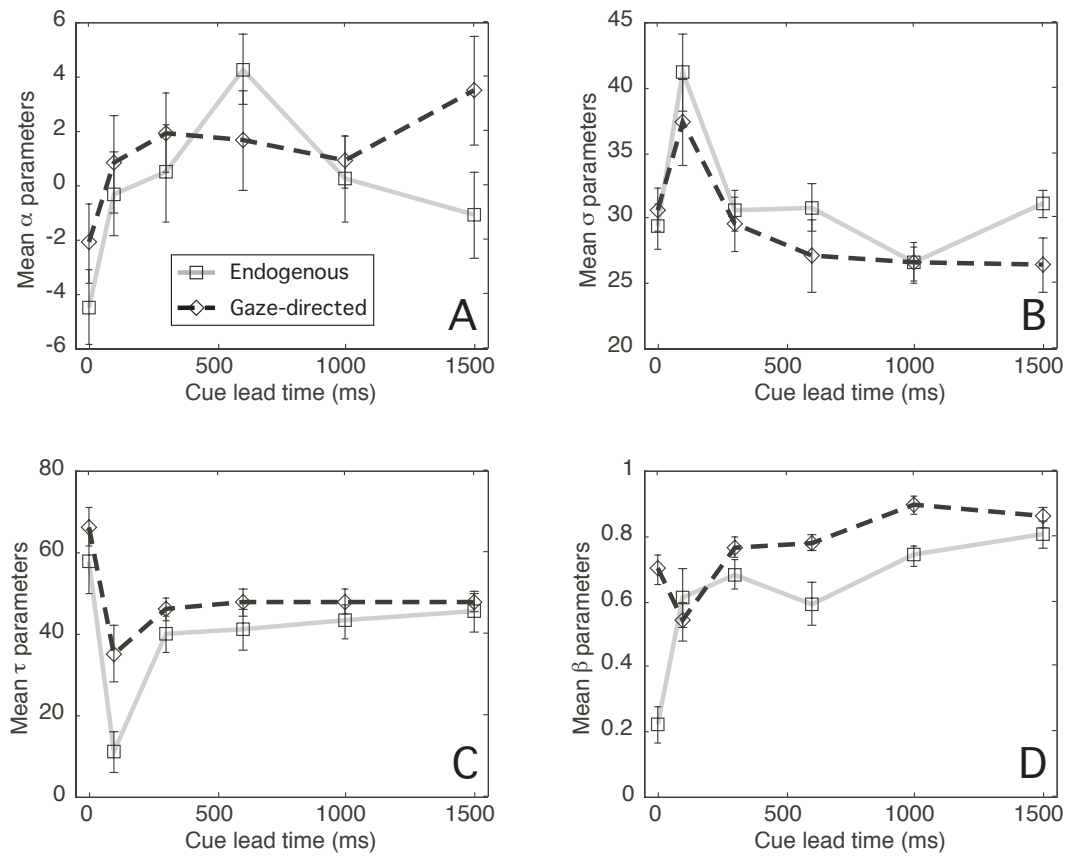
The peaks in  $\hat{\alpha}$  for the exogenous cue SJs are complimented by troughs in  $\hat{\tau}$  are approximately the same CLTs, as seen in Figure 18C. When the exogenous cues cause the maximum shift in  $\alpha$ , they also minimize  $\tau$  such that the observers are least likely to say that the two targets are simultaneous. The troughs reach their minimums around a CLT of 125 ms and increase to a baseline by the CLT of 500 ms.

Even though the model tests in Section V.4 above suggest that  $\alpha = 0$  for the TOJ, the shifts observed in the PSS in Figure 4 can be explained by changes in the response bias parameter  $\beta$ . As shown in Figure 18D,  $\hat{\beta}$  increases quickly and reaches a maximum by a CLT of about 125 ms, declining slightly thereafter for both the exogenous and isoluminant exogenous cues. Similarly, for the endogenous (Experiment 2) and gaze-directed (Experiment 3) cues, shifts in  $\beta$  can also explain the shifts in the PSSs in Figures 6 and 8, as  $\hat{\alpha}$ , shown in Figure 19A, is quite small for these experiments. As shown in Figure 19D,  $\hat{\beta}$  is non-significant at the earliest CLTs but increases to a plateau by a CLT of 300 ms.

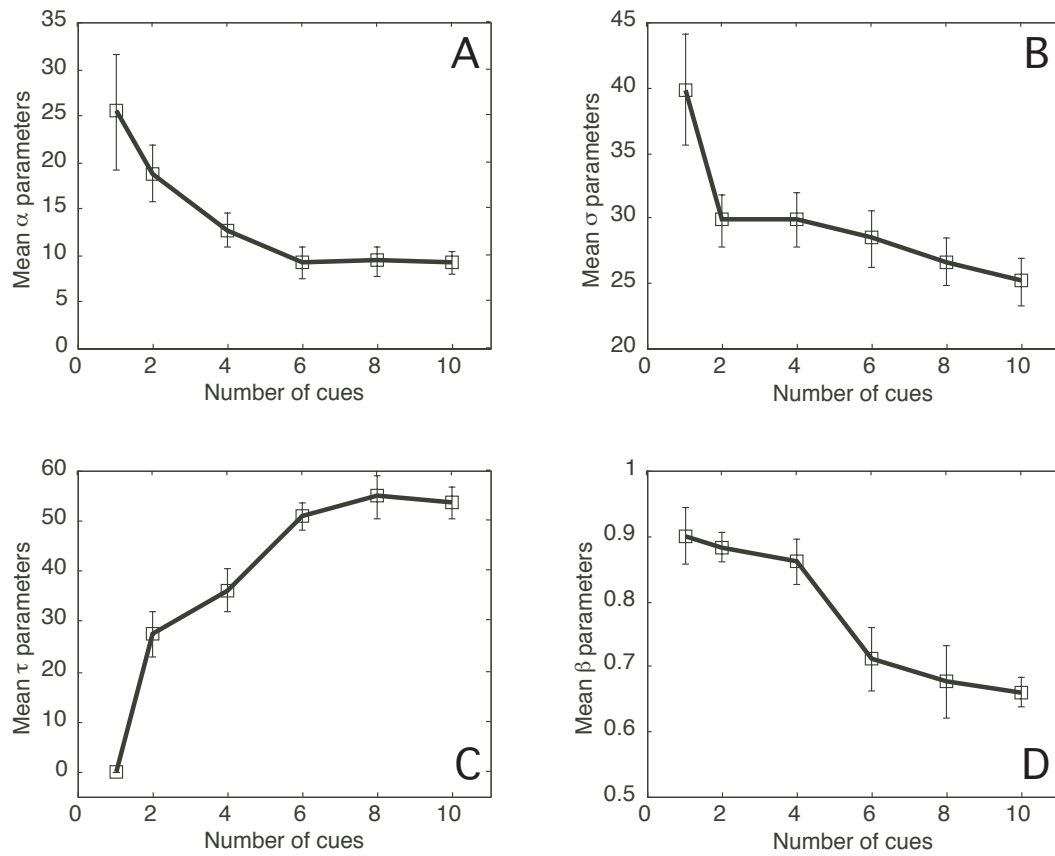
In Experiment 5, the decrease in  $\hat{\alpha}$ , shown in Figure 20A, with an increasing number of cues parallels the decrease in  $\hat{\beta}$ , shown in Figure 20D. The decrease in  $\hat{\alpha}$  seems to decelerate and appears to reach a non-zero asymptote by about 6 cues, while the decrease in  $\hat{\beta}$  does not obviously slow for a large number of cues, though it remains well above the unbiased 0.5.  $\hat{\tau}$  increases with an increasing number of cues, shown in Figure 20C, indicating that the observers are more willing to report the targets as simultaneous in the SJ task. If increasing the number of cues only affected the attention allocated to each, then both the stimuli and decision parameters can be said to depend on attention.



**Figure 18.** Best fit parameters for the triggered-moment model for exogenous cues in Experiments 1 and 4. A–D show the weighted mean across observers of the parameters that maximize the likelihood of the joint SJ and TOJ data. The error bars indicate the standard error of the weighted mean.



**Figure 19.** Best fit parameters for the triggered-moment model for non-exogenous cues in Experiments 2 and 3. A–D show the weighted means across observers of the parameters that maximize the likelihood of the joint SJ and TOJ data. The error bars indicate the standard error of the weighted mean.



**Figure 20.** Best fit parameters for the triggered-moment model for the multiple exogenous cues in Experiment 5. A–D show the weighted means across observers of the parameters that maximize the likelihood of the joint SJ and TOJ data. The error bars indicate the standard error of the weighted mean.

## VI General Discussion

There are several main points to be emphasized here. First, the observations of shifts in the PSS due to attentional manipulation in a TOJ are not sufficient evidence to accept the attentional acceleration (prior entry) hypothesis. Changes in response biases due to attention can also account for the shifts in the PSS without the inclusion of an attentional acceleration parameter, and it is impossible to discriminate the contributions of attentional acceleration from those of response bias using only a simple TOJ task. Experiments 1–5, which directed attention using exogenous, endogenous and gaze-directed cues, showed shifts in the PSS derived from the TOJ consistent with the time course of the type of attentional allocation used, but a triggered-moment model including a response bias parameter did not require an additional attentional acceleration parameter to explain the data.

The SJ is not subject to the same response biases and is better able to test the attentional acceleration hypothesis. A non-zero attentional acceleration parameter was required to explain the data obtained from the SJ in Experiments 1, 4 and 5 that used exogenous cues, but was not required to explain the SJ data from Experiments 2 or 3 that respectively used endogenous and gaze-directed cues. Endogenous attention does not seem to alter the latency of a stimulus, consistent with the physiological data that rarely showed any discernable change in latency due to endogenous cuing. Some latency effect might have been expected using gaze-directed cuing, however, based on Schuller and Rossion's (2001) recent result.

The essential difference between the types of cues that appeared to cause a latency reduction and those that did not was the presence of transient activity at the target location. The exogenous cues were abrupt onsets at the target location, while the endogenous and gaze-directed cues were remote from the target location.

Experiment 5 verified that a latency reduction for a stimulus still seemed to occur when one of multiple simultaneous exogenous cues occurred at its location. An attentional component to this effect was indicated by the decrease in the latency

reduction observed in Experiment 5 as the number of exogenous cues increased. However, this effect seemed to asymptote after about six cues. With a large number of cues, the attention allocated to any particular one should be small. The positive asymptote with a large number of cues is indicative of a non-attentional effect. Further evidence for this non-attentional effect comes from Experiments 1 and 4 in which the exogenous cues produced a significant latency reduction even when simultaneous with the target stimuli, too soon for the attentional effect to have developed.

The cause of the non-attentional effects produced by the exogenous cues is not clear. One possibility is that a sensory facilitation effect could lower the response thresholds of neural populations coding for the same retinotopic location. Alternatively, the visual system might be prone to confuse or be unable to encode the correct identities and timings of multiple temporal events occurring within a short time interval. Cai and Schlag (2001) demonstrated a dramatic example of such an identity misbinding between color and shape in a rapidly presented sequence of objects. A less dramatic time compression between the onsets of the cue and its target might also be sufficient. The largest observed latency effects on the order of 50 ms for a CLT of 125 ms, and Allik and Pulver (1994) showed that temporal order judgments seemed to be based on a low-pass temporal filter of the stimuli with a time constant of about 33 ms. Temporally precise recordings of neural activity or perhaps low-level neural simulations may be required to answer this question. A neural model might account for both the attentional and non-attentional effects of the exogenous cues.

The last point is that the triggered-moment model, with the inclusion of a response bias parameter, provides a good fit to the data. A four-parameter model that assumes the SJ and TOJ decision processes operate on the same latency distribution and even utilize the same  $\tau$  criterion is sufficient to explain the data for each condition in each experiment, and additional parameters are not warranted. This does not imply that the triggered-moment model describes the actual decision process in



the brain, but rather that the model and its assumptions provide a sufficient description of the data. Other models, such as the perceptual-moment model described in the Appendix, Section A.3, may provide an equally sufficient, or as preliminary results indicate for the perceptual-moment model in certain circumstances, perhaps a better descriptions.

In summary, many reported prior entry effects based on TOJs are likely to be caused by response biases. This would explain the conflicting results of the presence or absence of prior entry effects in historical experiments. It seems likely that the only true prior entry effects within the visual domain—attentional latency reductions that occur before the decision process—are stimulus-driven. However, since these sensory effects seem to include both attentional and non-attentional components, the explanatory power of the prior entry hypothesis is quite limited.

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## Appendix

### A.1 Derivatives of the likelihood function

For model comparison and also for certain minimization algorithms, it is necessary to

calculate the Hessian of  $\log \mathcal{L}$ . We have  $\mathbf{A} \equiv \nabla^2 \log \mathcal{L} \Rightarrow A_{jk} = \frac{\partial^2 \log \mathcal{L}}{\partial w_j \partial w_k}$ , with

$$\begin{aligned} \log \mathcal{L} &= \sum_{i=1}^m Y_i \log p_i + (n_i - Y_i) \log(1 - p_i) , \\ \frac{\partial \log \mathcal{L}}{\partial w_j} &= \sum_{i=1}^m \frac{\partial p_i}{\partial w_j} \left( \frac{Y_i}{p_i} - \frac{n_i - Y_i}{1 - p_i} \right) , \quad \text{and} \\ \frac{\partial^2 \log \mathcal{L}}{\partial w_j \partial w_k} &= \sum_{i=1}^m \frac{\partial^2 p_i}{\partial w_j \partial w_k} \left( \frac{Y_i}{p_i} - \frac{n_i - Y_i}{1 - p_i} \right) - \frac{\partial p_i}{\partial w_j} \frac{\partial p_i}{\partial w_k} \left[ \frac{Y_i}{p_i^2} + \frac{n_i - Y_i}{(1 - p_i)^2} \right] . \end{aligned}$$

## A.2 Derivatives of the triggered-moment model functions

### A.2.1 Simultaneity judgment

In this and subsequent sections, the following simplifying notation will be used:

$$\begin{aligned}\mathcal{N}^\pm &= \mathcal{N}(\pm\tau, \Delta t + \alpha, \sigma) & \mathcal{N}^0 &= \mathcal{N}(0, \Delta t + \alpha, \sigma) \\ \Phi^\pm &= \Phi(\pm\tau, \Delta t + \alpha, \sigma) & \Phi^0 &= \Phi(0, \Delta t + \alpha, \sigma) \\ \xi^\pm &= \pm\tau - \Delta t - \alpha & \xi^0 &= -\Delta t - \alpha\end{aligned}$$

The partial derivatives of  $p_i = \Phi^+ - \Phi^-$  are

$$\begin{aligned}\frac{\partial p_i}{\partial \alpha} &= -\mathcal{N}^+ + \mathcal{N}^- \\ \frac{\partial p_i}{\partial \sigma} &= -\frac{\xi^+}{\sigma} \mathcal{N}^+ + \frac{\xi^-}{\sigma} \mathcal{N}^- \\ \frac{\partial p_i}{\partial \tau} &= \mathcal{N}^+ + \mathcal{N}^- \\ \frac{\partial^2 p_i}{\partial \alpha^2} &= -\frac{\xi^+}{\sigma^2} \mathcal{N}^+ + \frac{\xi^-}{\sigma^2} \mathcal{N}^- \\ \frac{\partial^2 p_i}{\partial \alpha \partial \sigma} &= -\frac{1}{\sigma} \left( \frac{\xi^2}{\sigma^2} - 1 \right) \mathcal{N}^+ + \frac{1}{\sigma} \left( \frac{\xi^2}{\sigma^2} - 1 \right) \mathcal{N}^- \\ \frac{\partial^2 p_i}{\partial \alpha \partial \tau} &= \frac{\xi^+}{\sigma^2} \mathcal{N}^+ + \frac{\xi^-}{\sigma^2} \mathcal{N}^- \\ \frac{\partial^2 p_i}{\partial \sigma^2} &= -\frac{\xi^+}{\sigma^2} \left( \frac{\xi^2}{\sigma^2} - 2 \right) \mathcal{N}^+ + \frac{\xi^-}{\sigma^2} \left( \frac{\xi^2}{\sigma^2} - 2 \right) \mathcal{N}^- \\ \frac{\partial^2 p_i}{\partial \sigma \partial \tau} &= \frac{1}{\sigma} \left( \frac{\xi^2}{\sigma^2} - 1 \right) \mathcal{N}^+ + \frac{1}{\sigma} \left( \frac{\xi^2}{\sigma^2} - 1 \right) \mathcal{N}^- \\ \frac{\partial^2 p_i}{\partial \tau^2} &= -\frac{\xi^+}{\sigma^2} \mathcal{N}^+ + \frac{\xi^-}{\sigma^2} \mathcal{N}^-\end{aligned}$$

### A.2.2 Temporal order judgment

The partial derivatives of  $p_i = 1 + (\beta - 1)\Phi^+ - \beta\Phi^-$  are

$$\begin{aligned}
\frac{\partial p_i}{\partial \alpha} &= -(\beta - 1)\mathcal{N}^+ + \beta\mathcal{N}^- \\
\frac{\partial p_i}{\partial \sigma} &= -\frac{\beta - 1}{\sigma}\xi^+\mathcal{N}^+ + \frac{\beta}{\sigma}\xi^-\mathcal{N}^- \\
\frac{\partial p_i}{\partial \tau} &= (\beta - 1)\mathcal{N}^+ + \beta\mathcal{N}^- \\
\frac{\partial p_i}{\partial \beta} &= \Phi^+ - \Phi^- \\
\frac{\partial^2 p_i}{\partial \alpha^2} &= -\frac{\beta - 1}{\sigma^2}\xi^+\mathcal{N}^+ + \frac{\beta}{\sigma^2}\xi^-\mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \alpha \partial \sigma} &= -\frac{\beta - 1}{\sigma}\left(\frac{\xi^2}{\sigma^2} - 1\right)\mathcal{N}^+ + \frac{\beta}{\sigma}\left(\frac{\xi^2}{\sigma^2} - 1\right)\mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \alpha \partial \tau} &= \frac{\beta - 1}{\sigma^2}\xi^+\mathcal{N}^+ + \frac{\beta}{\sigma^2}\xi^-\mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \alpha \partial \beta} &= -\mathcal{N}^+ + \mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \sigma^2} &= -\frac{\beta - 1}{\sigma^2}\left(\frac{\xi^2}{\sigma^2} - 2\right)\xi^+\mathcal{N}^+ + \frac{\beta}{\sigma^2}\left(\frac{\xi^2}{\sigma^2} - 2\right)\xi^-\mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \sigma \partial \tau} &= \frac{\beta - 1}{\sigma}\left(\frac{\xi^2}{\sigma^2} - 1\right)\mathcal{N}^+ + \frac{\beta}{\sigma}\left(\frac{\xi^2}{\sigma^2} - 1\right)\mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \sigma \partial \beta} &= -\frac{\xi^+}{\sigma}\mathcal{N}^+ + \frac{\xi^-}{\sigma}\mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \tau^2} &= -\frac{\beta - 1}{\sigma^2}\xi^+\mathcal{N}^+ + \frac{\beta}{\sigma^2}\xi^-\mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \tau \partial \beta} &= \mathcal{N}^+ + \mathcal{N}^- \\
\frac{\partial^2 p_i}{\partial \beta^2} &= 0
\end{aligned}$$

### A.3 Perceptual moment theory

#### A.3.1 Introduction

The perceptual-moment theory is similar to the triggered moment theory in that two stimuli will be perceived as simultaneous if they occur within the same perceptual moment. However, unlike the triggered moment theory in which the perceptual moment begins at the onset of one of the stimuli, the perceptual-moment theory assumes that the timing of the perceptual moments is an ongoing process not related to any stimuli that might appear. The probability of two stimuli separated by  $\Delta t$  occurring within a perceptual moment of length  $\tau$  is  $1 - \frac{|\Delta t|}{\tau}$ .

#### A.3.2 Simultaneity judgment

The simultaneity decision is:

$$P(\text{“simultaneous”}) = \begin{cases} 1 - \frac{|\Delta L|}{\tau} & \text{if } |\Delta L| \leq \tau \\ 0 & \text{if } |\Delta L| > \tau. \end{cases} \quad (\text{A1})$$

Calculating the probabilities requires a convolution of the decision function and the latency distribution, and

$$\begin{aligned} P(\text{“simultaneous”}) &= \int_{-\tau}^{\tau} \left(1 - \frac{|x|}{\tau}\right) \mathcal{N}(x, \Delta t + \alpha, \sigma) dx \\ &= \frac{\sigma^2}{\tau} (\mathcal{N}^+ + \mathcal{N}^- - 2\mathcal{N}^0) + \frac{1}{\tau} (\xi^+ \Phi^+ + \xi^- \Phi^- - 2\xi^0 \Phi^0). \end{aligned} \quad (\text{A2})$$

### A.3.3 Order judgment

The temporal order decision rule for the perceptual-moment theory is:

$$P(\text{"}S_1 \text{ first"})=\begin{cases} 1 & \text{if } \Delta L > \tau \\ \frac{|\Delta L|}{\tau} + \beta \left(1 - \frac{|\Delta L|}{\tau}\right) & \text{if } 0 \leq \Delta L \leq \tau \\ \beta \left(1 - \frac{|\Delta L|}{\tau}\right) & \text{if } -\tau \leq \Delta L < 0 \\ 0 & \text{if } \Delta L < -\tau. \end{cases} \quad (\text{A3})$$

Therefore,

$$\begin{aligned} P(\text{"}S_1 \text{ first"})&= \int_{-\tau}^{\infty} \mathcal{N}(x, \Delta t + \alpha, \sigma) dx + \frac{1}{\tau} \int_0^{\tau} x \mathcal{N}(x, \Delta t + \alpha, \sigma) dx \\ &\quad + \beta \int_{-\tau}^{\tau} \left(1 - \frac{|x|}{\tau}\right) \mathcal{N}(x, \Delta t + \alpha, \sigma) dx \\ &= 1 + \frac{\beta - 1}{\tau} (\sigma^2 \mathcal{N}^+ + \xi^+ \Phi^+) + \frac{\beta}{\tau} (\sigma^2 \mathcal{N}^- + \xi^- \Phi^-) \\ &\quad - \frac{2\beta - 1}{\tau} (\sigma^2 \mathcal{N}^0 + \xi^0 \Phi^0). \end{aligned} \quad (\text{A4})$$

## A.4 Derivatives of the perceptual-moment theory functions

### A.4.1 Simultaneity judgment

The partial derivatives of  $p_i$  defined by Equation A2 are

$$\begin{aligned}
 \frac{\partial p_i}{\partial \alpha} &= -\frac{1}{\tau} (\Phi^+ + \Phi^- - 2\Phi^0) \\
 \frac{\partial p_i}{\partial \sigma} &= \frac{\sigma}{\tau} (\mathcal{N}^+ + \mathcal{N}^- - 2\mathcal{N}^0) \\
 \frac{\partial p_i}{\partial \tau} &= -\frac{\sigma^2}{\tau^2} (\mathcal{N}^+ + \mathcal{N}^- - 2\mathcal{N}^0) - \frac{\xi^0}{\tau^2} (\Phi^+ + \Phi^- - 2\Phi^0) \\
 \frac{\partial^2 p_i}{\partial \alpha^2} &= \frac{1}{\tau} (\mathcal{N}^+ + \mathcal{N}^- - 2\mathcal{N}^0) \\
 \frac{\partial^2 p_i}{\partial \alpha \partial \sigma} &= \frac{1}{\sigma \tau} (\xi^+ \mathcal{N}^+ + \xi^- \mathcal{N}^- - 2\xi^0 \mathcal{N}^0) \\
 \frac{\partial^2 p_i}{\partial \alpha \partial \tau} &= -\frac{1}{\tau} (\mathcal{N}^+ - \mathcal{N}^-) + \frac{1}{\tau^2} (\Phi^+ + \Phi^- - 2\Phi^0) \\
 \frac{\partial^2 p_i}{\partial \sigma^2} &= \frac{1}{\sigma^2 \tau} (\xi^+ \mathcal{N}^+ + \xi^- \mathcal{N}^- - 2\xi^0 \mathcal{N}^0) \\
 \frac{\partial^2 p_i}{\partial \sigma \partial \tau} &= -\frac{1}{\tau} \left( \frac{\xi^+}{\sigma} + \frac{\sigma}{\tau} \right) \mathcal{N}^+ + \frac{1}{\tau} \left( \frac{\xi^-}{\sigma} - \frac{\sigma}{\tau} \right) \mathcal{N}^- + \frac{2\sigma}{\tau^2} \mathcal{N}^0 \\
 \frac{\partial^2 p_i}{\partial \tau^2} &= \frac{1}{\tau} \left( \frac{2\sigma^2}{\tau^2} + 1 \right) (\mathcal{N}^+ + \mathcal{N}^-) - \frac{4\sigma^2}{\tau^3} \mathcal{N}^0 + \frac{2\xi^0}{\tau^3} (\Phi^+ + \Phi^- - 2\Phi^0)
 \end{aligned}$$



#### A.4.2 Temporal order judgment

The partial derivatives of  $p_i$  defined by Equation A4 are

$$\begin{aligned}
\frac{\partial p_i}{\partial \alpha} &= -\frac{\beta-1}{\tau} \Phi^+ - \frac{\beta}{\tau} \Phi^- + \frac{2\beta-1}{\tau} \Phi^0 \\
\frac{\partial p_i}{\partial \sigma} &= \frac{\beta-1}{\tau} \sigma \mathcal{N}^+ + \frac{\beta\sigma}{\tau} \mathcal{N}^- - \frac{2\beta-1}{\tau} \sigma \mathcal{N}^0 \\
\frac{\partial p_i}{\partial \tau} &= -\frac{\beta-1}{\tau^2} (\sigma^2 \mathcal{N}^+ + \xi^0 \Phi^+) - \frac{\beta}{\tau^2} (\sigma^2 \mathcal{N}^- + \xi^0 \Phi^-) + \frac{2\beta-1}{\tau^2} (\sigma^2 \mathcal{N}^0 + \xi^0 \Phi^0) \\
\frac{\partial p_i}{\partial \beta} &= \frac{\sigma^2}{\tau} (\mathcal{N}^+ + \mathcal{N}^- - 2\mathcal{N}^0) + \frac{1}{\tau} (\xi^+ \Phi^+ + \xi^- \Phi^- - 2\xi^0 \Phi^0) \\
\frac{\partial^2 p_i}{\partial \alpha^2} &= \frac{\beta-1}{\tau} \mathcal{N}^+ + \frac{\beta}{\tau} \mathcal{N}^- - \frac{2\beta-1}{\tau} \mathcal{N}^0 \\
\frac{\partial^2 p_i}{\partial \alpha \partial \sigma} &= \frac{\beta-1}{\sigma \tau} \xi^+ \mathcal{N}^+ + \frac{\beta \xi^-}{\sigma \tau} \mathcal{N}^- - \frac{2\beta-1}{\sigma \tau} \xi^0 \mathcal{N}^0 \\
\frac{\partial^2 p_i}{\partial \alpha \partial \tau} &= -\frac{\beta-1}{\tau^2} (\tau \mathcal{N}^+ - \Phi^+) + \frac{\beta}{\tau^2} (\tau \mathcal{N}^- + \Phi^-) - \frac{2\beta-1}{\tau^2} \Phi^0 \\
\frac{\partial^2 p_i}{\partial \alpha \partial \beta} &= -\frac{1}{\tau} (\Phi^+ + \Phi^- - 2\Phi^0) \\
\frac{\partial^2 p_i}{\partial \sigma^2} &= \frac{\beta-1}{\sigma^2 \tau} \xi^2 \mathcal{N}^+ + \frac{\beta}{\sigma^2 \tau} \xi^2 \mathcal{N}^- - \frac{2\beta-1}{\sigma^2 \tau} \xi^0 \mathcal{N}^0 \\
\frac{\partial^2 p_i}{\partial \sigma \partial \tau} &= -\frac{\beta-1}{\tau} \left( \frac{\xi^+}{\sigma} + \frac{\sigma}{\tau} \right) \mathcal{N}^+ + \frac{\beta}{\tau} \left( \frac{\xi^-}{\sigma} - \frac{\sigma}{\tau} \right) \mathcal{N}^- + \frac{2\beta-1}{\tau^2} \sigma \mathcal{N}^0 \\
\frac{\partial^2 p_i}{\partial \sigma \partial \beta} &= \frac{\sigma}{\tau} (\mathcal{N}^+ + \mathcal{N}^- - 2\mathcal{N}^0) \\
\frac{\partial^2 p_i}{\partial \tau^2} &= \frac{\beta-1}{\tau^3} [(2\sigma^2 + \tau^2) \mathcal{N}^+ + 2\xi^0 \Phi^+] + \frac{\beta}{\tau^3} [(2\sigma^2 + \tau^2) \mathcal{N}^- + 2\xi^0 \Phi^-] \\
&\quad - \frac{4\beta-2}{\tau^3} (\sigma^2 \mathcal{N}^0 + \xi^0 \Phi^0) \\
\frac{\partial^2 p_i}{\partial \tau \partial \beta} &= -\frac{\sigma^2}{\tau^2} (\mathcal{N}^+ + \mathcal{N}^- - 2\mathcal{N}^0) - \frac{\xi^0}{\tau^2} (\Phi^+ + \Phi^- - 2\Phi^0) \\
\frac{\partial^2 p_i}{\partial \beta^2} &= 0
\end{aligned}$$